

Chapter 1 Heating processes

Section 1.1 Heat and temperature

Worked example: Try yourself 1.1.1

CALCULATING THE CHANGE IN INTERNAL ENERGY

A student places a heating element and a paddle-wheel apparatus in an insulated container of water. She calculates that the heating element transfers 2530J of heat energy to the water and the paddle does 240J of work on the water. Calculate the change in internal energy of the water.

Thinking	Working
Heat is added to the system, so Q is positive. Work is done on the system, so W is positive.	$\Delta U = Q + W$ $= (+2530) + (+240)$
Note that the units are J, so express the final answer in J.	$\Delta U = 2770\text{J}$

Section 1.1 Review

KEY QUESTIONS SOLUTIONS

- C. The kinetic particle theory states that the particles in all substances (regardless of their state) are in constant motion.
- The chicken and the air in the oven are not in thermal equilibrium.
 - Thermal energy flows from the hot air into the chicken.
 - The chicken and the air in the oven are in thermal equilibrium.
- C and D. Negative kelvin values and Celsius values below -273 are not possible because temperatures below absolute zero are not possible.
- The temperature of the gas is just above absolute zero so the particles have very little energy.
- $K = ^\circ\text{C} + 273$
 $= 30 + 273$
 $= 303\text{K}$
 - $^\circ\text{C} = K - 273$
 $= 375 - 273$
 $= 102^\circ\text{C}$
- 300K is 27°C . Higher temperatures mean molecules have greater average kinetic energy. So the average kinetic energy of the hydrogen particles in tank B is greater than the average kinetic energy of the hydrogen particles in tank A.
- absolute zero, 10K, -180°C , 100K, freezing point of water
- $\Delta U = Q + W$
 $= -20 + -50$
 $= -70\text{kJ}$
- $\Delta U = Q + W$
 $= 75 + 150$
 $= 225\text{J}$
- $\Delta U = Q + W$
 $250 = -300 + W$
 $W = 550\text{J}$
The scientist does 550J of work on the sodium.

Section 1.2 Specific heat capacity

Worked example: Try yourself 1.2.1

CALCULATIONS USING SPECIFIC HEAT CAPACITY

A bath contains 75 L of water. Initially the water is at 50°C. Calculate the amount of energy that must be transferred from the water to cool the bath to 30°C.	
Thinking	Working
Calculate the mass of water. 1 L of water = 1 kg	Volume = 75 L So mass of water = 75 kg
$\Delta T = \text{final temperature} - \text{initial temperature}$	$\Delta T = 30 - 50 = -20^\circ\text{C}$
From Table 1.2.1, $c_{\text{water}} = 4180 \text{ J kg}^{-1} \text{ K}^{-1}$. Use the equation $Q = mc\Delta T$.	$Q = mc\Delta T$ $= 75 \times 4180 \times 20$ $= 6\,270\,000$ $= 6.27 \times 10^6 \text{ J transferred from the water}$

Worked example: Try yourself 1.2.2

COMPARING SPECIFIC HEAT CAPACITIES

What is the ratio of the specific heat capacity of liquid water to that of steam?	
Thinking	Working
Table 1.2.1 has the specific heat capacities of water in different states.	$c_{\text{water}} = 4180 \text{ J kg}^{-1} \text{ K}^{-1}$ $c_{\text{steam}} = 2000 \text{ J kg}^{-1} \text{ K}^{-1}$
Divide the specific heat of water by the specific heat of steam.	Ratio = $\frac{c_{\text{water}}}{c_{\text{steam}}}$ $= \frac{4180}{2000}$
Note that ratios have no units since the unit of each quantity is the same and cancels out.	Ratio ≈ 2.1

Section 1.2 Review

KEY QUESTIONS SOLUTIONS

- Water requires more energy per degree Celsius heated because the specific heat capacity of water is much greater than that of aluminium.
- All other variables being the same, as aluminium has the highest value for specific heat capacity, it will contain the most thermal energy.
- 100 mL of water has a mass of 0.1 kg.
 $Q = mc\Delta T$
 $= 0.1 \times 4180 \times (20 - 15)$
 $= 2090 \text{ J}$
- 150 mL of water has a mass of 0.15 kg.
 $Q = mc\Delta T$
 $= 0.15 \times 4180 \times (50 - 10)$
 $= 25080 \text{ J or } 25.1 \text{ kJ}$

- 5 Remember that both ΔT and mass are proportional to energy.
Use the relationship $Q = mc\Delta T$.
If $x = 10c$ then $20c = 2xJ$.
- 6 The ratio of the temperature rise is equal to the inverse ratio of the specific heat capacities as $\Delta T = \frac{1}{c}$
Ratio of temperature rise = $\frac{4180}{900} = 4.64$
The temperature of the aluminium is 4.64 times that of the water.
- 7 B. Different states will have different specific heat capacities.
- 8 If 4.0kJ of energy is required to raise the temperature of 1.0kg of paraffin by 2.0°C, then 2.0kJ of energy is required to raise the temperature of 1.0kg of paraffin by 5.0°C.
So to raise the temperature of 5.0kg, you will need five times as much energy, i.e. $5 \times 2.0\text{kJ} = 10\text{kJ}$ to raise the temperature of 5.0kg of paraffin by 1.0°C.
Mathematically: Calculate c for paraffin, so $c = \frac{Q}{m\Delta T} = \frac{4000}{1 \times 2} = 2000\text{Jkg}^{-1}\text{K}^{-1}$
Then for 5.0kg, $Q = mc \Delta T = 5.0 \times 2090 \times 1.0 = 10\text{kJ}$
- 9 $Q = mc \Delta T$
 $10\,500 = 0.25 \times 4180 \times (T - 20)$
 $10 = T - 20$
 $T = 30^\circ\text{C}$
Final temperature = 30°C
- 10 $Q = mc \Delta T$
 $-13\,200 = m \times 440 \times -30$
 $m = -13\,200 \div -13\,200$
 $= 1\text{kg}$

Section 1.3 Latent heat

Worked example: Try yourself 1.3.1

LATENT HEAT OF FUSION

How much energy must be removed from 5.5 kg of liquid lead at 327°C to produce a block of solid lead at 327°C ? Express your answer in kJ.	
Thinking	Working
Cooling from liquid to solid involves the latent heat of fusion, where the energy is removed from the lead. Use Table 1.3.1 to find the latent heat of fusion for lead.	$L_{\text{fusion}} = 0.25 \times 10^5\text{Jkg}^{-1}$
Use the equation: $Q = mL_{\text{fusion}}$	$Q = mL_{\text{fusion}}$ $= 5.5 \times 0.25 \times 10^5$ $= 1.4 \times 10^5\text{J}$
Convert to kJ.	$Q = 1.4 \times 10^2\text{kJ}$

Worked example: Try yourself 1.3.2
CHANGE IN TEMPERATURE AND STATE

3 L of water is heated from a fridge temperature of 4°C to its boiling point at 100°C. It is boiled at this temperature until it is completely evaporated. How much energy in total is required to raise the temperature and boil the water?	
Thinking	Working
Calculate the mass of water involved.	3 L of water = 3 kg
Find the specific heat capacity of water from Table 1.2.1.	$c = 4180 \text{ J kg}^{-1} \text{ K}^{-1}$
Use the equation $Q = mc\Delta T$ to calculate the heat energy required to change the temperature of water from 4°C to 100°C.	$Q = mc\Delta T$ $= 3 \times 4180 \times (100 - 4)$ $= 1\,203\,800 \text{ J}$
Find the specific latent heat of vaporisation of water.	$L_{\text{vapour}} = 22.5 \times 10^5 \text{ J kg}^{-1}$
Use the equation $Q = mL_{\text{vapour}}$ to calculate the latent heat required to boil water.	$Q = mL_{\text{vapour}}$ $= 3 \times 22.5 \times 10^5$ $= 6\,750\,000 \text{ J}$
Find the total energy required to raise the temperature and change the state of the water.	$\text{Total } Q = 1\,203\,800 + 6\,750\,000$ $= 8 \times 10^6 \text{ J}$

Section 1.3 Review

KEY QUESTIONS SOLUTIONS

1 The mercury is changing state from solid to liquid. It is melting; temperature does not change during phase transitions as the average kinetic energy does not change.

2 -39°C

3 357°C

4 $Q = mL_{\text{fusion}}$
 $126 = 0.01 \times L_{\text{fusion}}$
 $L_{\text{fusion}} = 126 \div 0.01 = 12\,600$
 $= 1.26 \times 10^4 \text{ J kg}^{-1}$

5 $Q = mL_{\text{vapour}}$
 $3520 - 670 = 0.01 \times L_{\text{vapour}}$
 $L_{\text{vapour}} = 2850 \div 0.01 = 285\,000$
 $= 2.85 \times 10^5 \text{ J kg}^{-1}$

6 $Q = mL_{\text{fusion}}$
 $= 0.1 \times 22.5 \times 10^5$
 $= 2.25 \times 10^5 \text{ J}$

7 Energy needed to raise the ice from -4.00°C to 0°C

$$Q = mc\Delta T$$

$$= 0.100 \times 2100 \times 4.00$$

$$= 840 \text{ J}$$

Energy needed to melt the ice at 0°C

$$Q = mL_{\text{fusion}}$$

$$= 0.100 \times 3.34 \times 10^5$$

$$= 3.34 \times 10^4 \text{ J}$$

$$\text{Total energy} = 840 + (3.34 \times 10^4)$$

$$= 3.42 \times 10^4$$

$$= 34 \text{ kJ}$$

- 8 Hot water molecules have more energy than cold water molecules and will be able to leave the surface of the spa-pool water at a greater rate than cold water.
- 9 Most of the liquid has evaporated and the remaining liquid becomes colder as it does so, which in turn cools the floor.

Section 1.4 Heating and cooling

Worked example: Try yourself 1.4.1

CALCULATING THERMAL EQUILIBRIUM 1

4.00 kg of water initially at 85.0°C is mixed with 3.00 kg of water initially at 25.0°C. What is the final temperature of the water once thermal equilibrium is reached?	
Thinking	Working
Total energy lost by hot water = total energy gained by cold water That is, the energy change, ΔQ , is equal for the hot and cold water. Use $\Delta Q = mc\Delta T$ Assume no loss to the surrounding environment.	$\Delta Q_{\text{hot}} = \Delta Q_{\text{cold}}$ $m_{\text{hot}}c\Delta T_{\text{hot}} = m_{\text{cold}}c\Delta T_{\text{cold}}$
Since specific heat capacity of the water will be the same on both sides of the equation, the equation can be simplified.	$m_{\text{hot}}\Delta T_{\text{hot}} = m_{\text{cold}}\Delta T_{\text{cold}}$
Substitute the known values and simplify for the equilibrium temperature, T .	$4.00 \times (85.0 - T) = 3.00 \times (T - 25.0)$ $340 - 4.00T = 3.00T - 75.0$ $340 + 75.0 = 3.00T + 4.00T$ $415 = 7.00T$ $T = \frac{415}{7.00}$ $T = 59.3^{\circ}\text{C}$
Do a quick intuitive check. Does the answer make sense?	As most of water was warmer, the final temperature should be closer to the temperature of the original warmer water than to the original cooler water.

Worked example: Try yourself 1.4.2
CALCULATING THERMAL EQUILIBRIUM 2

<p>A 75.0g piece of copper is heated over a flame for several minutes. The copper is then plunged into an insulated, closed container containing 0.500L of cool water, originally at 20.0°C. When thermal equilibrium is reached, the temperature of the water is found to be 22.0°C. If no water changes state to become steam and there are no other energy losses, then what was the temperature of the copper just before it was immersed in the water?</p>	
Thinking	Working
Convert all masses to standard units (kg).	Mass of copper = 75.0g = 0.0750 kg Mass of water = 0.500 kg (1.00L of water = 1.00 kg)
Refer to Table 1.2.1 for the relevant specific heat capacity (c) values.	$c_{\text{copper}} = 390 \text{ Jkg}^{-1} \text{ K}^{-1}$ $c_{\text{water}} = 4180 \text{ Jkg}^{-1} \text{ K}^{-1}$
Total energy lost by copper = total energy gained by water That is, the energy change, ΔQ , is equal for the copper and the water.	$\Delta Q_{\text{copper}} = \Delta Q_{\text{water}}$ $m_c c_c \Delta T_c = m_w c_w \Delta T_w$
Substitute the known values, expand and simplify to solve for the initial temperature of the copper.	$m_c c_c \Delta T_c = m_w c_w \Delta T_w$ $0.0750 \times 390 \times (T_c - 22.0) = 0.500 \times 4180 \times (22.0 - 20.0)$ $29.25T_c - 643.5 = 4180$ $29.25T_c = 4823.5$ $T_{\text{copper}} = \frac{4823.5}{29.25}$ $T_{\text{copper}} = 165^\circ\text{C}$

Worked example: Try yourself 1.4.3
CHANGES OF STATE

Calculate the heat energy that must be lost, in J, to convert 5.00 kg of water vapour at 140.0°C into solid ice at 0.00°C.	
Thinking	Working
Identify the steps involved in the process	Step 1: Steam at 140.0°C to steam at 100.0°C Step 2: Steam at 100.0°C to water at 100.0°C Step 3: Water at 100.0°C to water at 0.00°C Step 4: Water at 0.00°C to ice at 0.00°C
Identify values for L and c for each step. Use tables 1.2.1, 1.3.1 and 1.3.2 to look up the values.	$c_{\text{steam}} = 2000 \text{ Jkg}^{-1} \text{ K}^{-1}$ $c_{\text{water}} = 4180 \text{ Jkg}^{-1} \text{ K}^{-1}$ $L_{\text{fusion}} = 3.34 \times 10^5 \text{ Jkg}^{-1}$ $L_{\text{vapour}} = 22.5 \times 10^5 \text{ Jkg}^{-1}$
Calculate the energy required for each step separately using the appropriate equation for specific heat or latent heat.	Step 1: Cooling the steam $Q_1 = mc\Delta T$ $= 5.00 \times 2000 \times 40.0$ $= 4.00 \times 10^5 \text{ J}$ Step 2: Condensing the steam $Q_2 = mL_{\text{vapour}}$ $= 5.00 \times 22.5 \times 10^5$ $= 1.125 \times 10^7 \text{ J}$ Step 3: Cooling the water $Q_3 = mc\Delta T$ $= 5.00 \times 4180 \times 100.0$ $= 2.09 \times 10^6 \text{ J}$ Step 4: Freezing the water $Q_4 = mL_{\text{fusion}}$ $= 5.00 \times 3.34 \times 10^5$ $= 1.67 \times 10^6 \text{ J}$
Add the energy required for each step together to find the total energy required.	$Q_T = Q_1 + Q_2 + Q_3 + Q_4$ $= (4.00 \times 10^5) + (1.125 \times 10^7) + (2.09 \times 10^6) + (1.67 \times 10^6)$ $= 1.54 \times 10^7 \text{ J}$

Section 1.4 Review

KEY QUESTIONS SOLUTIONS

1 Aluminium. All other variables being the same, aluminium has the highest value for specific heat capacity so it will absorb the highest amount of thermal energy.

2 $T = 20.6^\circ\text{C}$

$$Q_{\text{lost by hot}} = Q_{\text{gained by cold}}$$

$$m_h c \Delta T = m_c c \Delta T$$

$$m_h \Delta T = m_c \Delta T$$

$$10.0 \times (65.0 - T) = 80.0 \times (T - 15.0)$$

$$650.0 - 10.0T = 80.0T - 1200.0$$

$$90.0T = 1850.0$$

$$T = \frac{1850.0}{90.0}$$

$$= 20.56^\circ\text{C}$$

$$= 20.6^\circ\text{C} \text{ (to 3 significant figures)}$$

3 $Q_{\text{lost by copper}} = Q_{\text{gained by water}}$

$$m_c c_c \Delta T = m_w c_w \Delta T$$

$$20.0 \times 390 \times (100.0 - T) = 5.00 \times 4180 \times (T - 20.0)$$

$$7.80 \times 10^5 - 7.80 \times 10^3 T = 2.09 \times 10^4 T - 4.18 \times 10^5$$

$$2.87 \times 10^4 T = 1.198 \times 10^6$$

$$T = \frac{1.198 \times 10^6}{2.87 \times 10^4}$$

$$= 41.7^\circ\text{C}$$

4 $Q_{\text{cooling}} = Q_{\text{warming}}$

$$m c_{\text{water}} \Delta T = m c_{\text{water}} \Delta T$$

$$m_{\text{cooled}} \Delta T = m_{\text{heated}} \Delta T$$

$$m_{\text{cooled}} \times (45.0 - 36.0) = 12.0 \times (36.0 - 19.0)$$

$$m_{\text{cooled}} \times 9.0 = 204$$

$$m_{\text{cooled}} = \frac{204}{9.0}$$

$$= 22.7 \text{ kg}$$

5 $Q_{\text{lost by iron}} = Q_{\text{gain by water}}$

$$m_i c_i \Delta T = m_w c_w \Delta T$$

$$598 \times 440 \times (1250 - T) = 938 \times 4180 \times (T - 21.0)$$

$$3.289 \times 10^8 - 2.631 \times 10^5 T = 3.9208 \times 10^6 T - 8.2338 \times 10^7$$

$$4.1839 \times 10^6 T = 4.1124 \times 10^8$$

$$T = \frac{4.1124 \times 10^8}{4.1839 \times 10^6}$$

$$= 98.3^\circ\text{C}$$

6 $Q_{\text{iron}} + Q_{\text{aluminium}} = Q_{\text{water}}$

$$m_i c_i \Delta T + m_a c_a \Delta T = m_w c_w \Delta T$$

$$10.0 \times 440 \times (20.0 - T) + 10.0 \times 900 \times (20.0 - T) = 100 \times 4180 \times (T - 12.0)$$

$$8.80 \times 10^4 - 4.40 \times 10^3 T + 1.80 \times 10^5 - 9.00 \times 10^3 T = 4.18 \times 10^5 T - 5.016 \times 10^6$$

$$4.18 \times 10^5 T + 4.40 \times 10^3 T + 9.00 \times 10^3 T = 8.80 \times 10^4 + 1.80 \times 10^5 + 5.016 \times 10^6$$

$$4.314 \times 10^5 T = 5.284 \times 10^6$$

$$T = \frac{5.284 \times 10^6}{4.314 \times 10^5}$$

$$= 12.2^\circ\text{C}$$

- 7 $Q_{\text{total}} = Q_{\text{heating}} + Q_{\text{steam}}$
 $= mc\Delta T + mL_v$
 $= 50.0 \times 4180 \times (100 - 20.0) + 50.0 \times 2.25 \times 10^6$
 $= 1.672 \times 10^7 + 1.125 \times 10^8$
 $= 1.29 \times 10^8 \text{ J}$
- 8 $Q_{\text{steam}} + Q_{\text{water}} = Q_{\text{potatoes}}$
 $m_s L_v + m_s c_w \Delta T = m_p c_p \Delta T$
 $m_s \times 2.25 \times 10^6 + m_s \times 4180 \times (100 - 85.0) = 3.00 \times 3430 \times (85.0 - 12.5)$
 $2.25 \times 10^6 m_s + 6.27 \times 10^4 m_s = 7.46 \times 10^5$
 $2.3127 \times 10^6 m_s = 7.46 \times 10^5$
 $m_s = \frac{7.46 \times 10^5}{2.3127 \times 10^6}$
 $= 0.323 \text{ kg}$
- 9 $Q_{\text{total}} = mc\Delta T + mL_{\text{fusion}}$
 $= 1.25 \times 233 \times (961 - 20.0) + 1.25 \times 1.11 \times 10^3$
 $= 2.74 \times 10^5 + 1.39 \times 10^3$
 $= 2.75 \times 10^5 \text{ J}$
- 10 $Q_{\text{total}} = mc_s \Delta T + mL_v + mc_w \Delta T$
 $= 0.755 \times 2000 \times (110 - 100) + 0.755 \times 2.25 \times 10^6 + 0.755 \times 4180 \times (100 - 25.0)$
 $= 1.51 \times 10^4 + 1.70 \times 10^6 + 2.367 \times 10^5$
 $= 1.95 \times 10^6 \text{ J}$

CHAPTER 1 REVIEW

- A. The kinetic theory states that particles are in constant motion.
- Temperature—the average kinetic energy of particles in a substance.
- Heat refers to the energy that is transferred between objects, whereas temperature is a measure of the average kinetic energy of the particles within a substance.
- a $5 + 273 = 278 \text{ K}$
 b $200 - 273 = -73^\circ \text{C}$
- The fixed points must be reproducible under any conditions. The starting point of the scale must be zero, with no negative values.
- 0°C is not the lowest value on the Celsius scale—negative values are possible. The freezing and boiling points of water are not fixed but vary with changing pressure.
- As thermal equilibrium is reached, the balls must be at the same temperature.
- B
- The substance is changing state—in this case, it is melting. The heat energy is used to increase the potential energy of the particles in the solid instead of increasing their kinetic energy, so the temperature does not change. The energy needed to change from solid to liquid is the latent heat of fusion.
- Both have the same kinetic energy as their temperatures are the same; however, the steam has more potential energy due to its change in state. Therefore the steam has greater internal energy.
- The higher energy particles are escaping, leaving behind the lower energy particles. The result is that the average kinetic energy of the remaining particles decreases, thus the temperature drops.
- $Q = mc\Delta T$
 $c = \frac{Q}{m\Delta T}$
 $= \frac{5020}{2.00 \times 20}$
 $= 125.5 \text{ J kg}^{-1} \text{ K}^{-1}$
 $= 126 \text{ J kg}^{-1} \text{ K}^{-1}$

$$13 \quad Q = mL$$

$$= 0.08 \times 0.88 \times 10^5$$

$$= 7.0 \text{ kJ}$$

- 14 $c_{\text{copper}} = 390 \text{ J kg}^{-1} \text{ K}^{-1}$, $c_{\text{iron}} = 440 \text{ J kg}^{-1} \text{ K}^{-1}$. Copper requires less thermal energy to heat it than iron so will cool the water travelling through it less than iron. However, it is also a better conductor of heat so will require additional insulation to avoid transferring more heat to the surrounds.

$$15 \quad \Delta U = Q - W$$

$$= +14600 - (-2.65 \times 10^6)$$

$$= 2664600 \text{ J}$$

$$Q = mL_{\text{fusion}} + mc\Delta T$$

$$2664600 = 4.55 \times 3.34 \times 10^5 + 4.55 \times 4180 \times (T - 0)$$

$$T = 60^\circ\text{C}$$

- 16 Note that both the cup and the water must be cooled since there will be heat transfer between the two materials in contact.

$$Q_{\text{melting ice}} + Q_{\text{heating ice}} = Q_{\text{cooling water}} + Q_{\text{cooling cup}}$$

$$m_{\text{ice}}L_f + m_{\text{ice}}c_{\text{water}}\Delta T = m_{\text{water}}c_{\text{water}}\Delta T + m_{\text{copper}}c_{\text{copper}}\Delta T$$

$$m_{\text{ice}} \times 3.34 \times 10^5 + m_{\text{ice}} \times 4180 \times (20 - 0) = 0.100 \times 4180 \times (60 - 20) + 0.200 \times 390 \times (60 - 20)$$

$$3.34 \times 10^5 m_{\text{ice}} + 8.36 \times 10^4 m_{\text{ice}} = 1.672 \times 10^4 + 3120$$

$$4.176 \times 10^5 m_{\text{ice}} = 1.984 \times 10^4$$

$$m_{\text{ice}} = \frac{1.984 \times 10^4}{4.176 \times 10^5}$$

$$= 4.75 \times 10^{-2} \text{ kg}$$

$$17 \quad Q_{\text{milk}} = Q_{\text{steam}} + Q_{\text{water}}$$

$$m_m c \Delta T = m_s L_f + m_w c_w \Delta T$$

$$0.425 \times 3930 \times (70.0 - 4.00) = 2.25 \times 10^6 m + m \times 4180 \times (100 - 70.0)$$

$$1.10 \times 10^5 = 2.25 \times 10^6 m + 1.254 \times 10^5 m$$

$$2.375 \times 10^6 m = 1.10 \times 10^5$$

$$m = \frac{1.10 \times 10^5}{2.375 \times 10^6}$$

$$= 4.63 \times 10^{-2} \text{ kg}$$

$$18 \quad Q_{\text{lemon}} = Q_{\text{ice}} + Q_{\text{water}}$$

$$m_l c \Delta T = m_i L_f + m_w c_w \Delta T$$

$$0.468 \times 3850 \times (20.0 - 3.00) = 3.34 \times 10^5 m + m \times 4180 \times (3.00 - 0.00)$$

$$3.063 \times 10^4 = 3.34 \times 10^5 m + 1.254 \times 10^4 m$$

$$3.063 \times 10^4 = 3.465 \times 10^5 m$$

$$m = \frac{3.063 \times 10^4}{3.465 \times 10^5}$$

$$= 8.84 \times 10^{-2} \text{ kg}$$

$$19 \quad Q_{\text{steam}} = Q_{\text{ice}}$$

$$m_s c_s \Delta T + m_s L_v + m_s c_w \Delta T = m_i c_i \Delta T + m_i L_f + m_i c_w \Delta T$$

$$m_s \times 2000 \times (115 - 100) + 2.25 \times 10^6 m_s + m_s \times 4180 \times (100 - 55.0) = 2.50 \times 2100 \times (0 - (-12.5)) + 2.50 \times 3.34 \times 10^5 + 2.50 \times 4180 \times (55.0 - 0)$$

$$3.00 \times 10^4 m_s + 2.25 \times 10^6 m_s + 1.881 \times 10^5 m_s = 6.56 \times 10^4 + 8.35 \times 10^5 + 5.7475 \times 10^5$$

$$2.4681 \times 10^6 m_s = 1.4754 \times 10^6$$

$$m_s = \frac{1.4754 \times 10^6}{2.4681 \times 10^6}$$

$$= 0.598 \text{ kg}$$

20

$$Q_{\text{iron}} = Q_{\text{water}}$$

$$m_i c_i \Delta T = m_w c_w \Delta T + m_w L_v$$

$$18.0 \times 440 \times (545 - T) = 1.50 \times 4180 \times (100 - 22.0) + 1.50 \times 2.25 \times 10^6$$

$$4.316 \times 10^6 - 7.920 \times 10^3 T = 4.8906 \times 10^5 + 3.375 \times 10^6$$

$$7.920 \times 10^3 T = 4.316 \times 10^6 - 3.8641 \times 10^6$$

$$T = \frac{4.519 \times 10^5}{7.920 \times 10^3}$$

$$= 57.1^\circ\text{C}$$

Chapter 2 Moving heat around

Section 2.1 Heat and temperature

Worked example: Try yourself 2.1.1

ENERGY EFFICIENCY

An electric kettle uses 23.3 kJ of electrical energy as it boils a quantity of water. The efficiency of the kettle is 18%. How much electrical energy is used in actually boiling the water? Give your answer in kJ.	
Thinking	Working
Recall the formula for efficiency of energy transfers.	efficiency (η) = $\frac{\text{energy output}}{\text{energy input}} \times 100\%$
Substitute the known values into the formula.	input = 23.2 kJ efficiency = 18% $18 = \frac{\text{output}}{23.3 \times 10^3} \times 100$
Solve the equation to find the unknown.	output = $\frac{18 \times 23.3 \times 10^3}{100}$ = 4190 J = 4.19 kJ

Section 2.1 Review

KEY QUESTIONS SOLUTIONS

- In coal-fired generators, the *chemical* energy from the coal is used to change water into steam, which possesses *heat* energy. The steam drives a turbine, which produces *kinetic* energy, which drives a generator, which produces *electrical* energy.
- the mechanical work done on the water
- $W = Fs$
= $4.5 \times 9.80 \times 6.0$
= 265 J
- $\Delta U = Q + W$
= $2530 + 240$
= 2770 J
- $W = Fs = mg \times s$
= 980×2.4
= 2352 J
= 2.4 kJ
- 0 J. Since there is no change in position when the mass is being held steady, no work is done.
- $\eta = \frac{\text{output}}{\text{input}} \times 100\%$
= $\frac{1.2}{4.8} \times 100$
= 25%
- $\eta = \frac{\text{output}}{\text{input}} \times 100\%$
 $70 = \frac{\text{output}}{3.6 \times 10^3} \times 100\%$
output = $\frac{70}{100} \times 3.6 \times 10^3$
= 2520 J
= 2.5 kJ (to two significant figures)

$$\begin{aligned}
 9 \quad U &= Q + W \\
 &= -1239 + 845 \\
 &= -394 \text{ J} \\
 10 \quad \eta &= \frac{\text{output}}{\text{input}} \times 100 \\
 30 &= \frac{\text{output}}{2000} \times 100 \\
 \text{output} &= \frac{30}{100} \times 2000 \\
 &= 600 \text{ J}
 \end{aligned}$$

Section 2.2 Conduction

KEY QUESTIONS SOLUTIONS

- The process is quite slow because the mass of the particles is relatively large and the vibrational velocities are fairly low.
- Metals conduct heat by free electrons as well as by molecular collisions. Wood does not have any free electrons, so it is a poor conductor of heat.
- The thickness, surface area, nature of the material and the temperature difference between it and another material.
- Copper is a better conductor of heat than stainless steel.
- C. Air is a poor conductor of heat so it limits the transfer of heat.
- A lot of air is trapped in the down. As air is a poor conductor of heat, the down-filled quilt limits the transfer of heat away from the person.
- The insulation batts stop the thermal energy from escaping the house. The air trapped in the batts causes the insulation to have low conductivity and so the thermal energy is not able to escape from the house.
- Plastic and rubber have low conductivity, so they do not allow the transfer of heat from your hand very easily. Metal has high conductivity, so heat transfers from your hand easily and your hand feels cold.
- Living areas with large windows should be on the northern side of the house and bedrooms with small windows should be on the southern side.
- The main difference between single-glazed and double-glazed windows is that double-glazed windows have twin panes of glass with a sealed air space between them, which provides additional thermal resistance.

Section 2.3 Convection

KEY QUESTIONS SOLUTIONS

- liquids and gases
- upwards
- Air over certain places, such as roads, heats up and as a result becomes less dense. The less dense air rises, forming a column of rising air called a thermal.
- Liquids and gases can transfer heat quite quickly through convection, but they are both poor conductors of thermal energy.
- It is not possible for solids to pass on heat by convection because solids do not contain the free molecules that are required to establish convection currents.
- The source of heat, the Sun, is at the top of the water. It takes much longer to heat a liquid when the source is at the top as the convection currents will also remain near the top. The warm water is less dense than the cool water and will not allow convection currents to form throughout the water.
- Near the heat source, gas or liquid expands and hence become less dense. The less-dense liquid or gas rises, while cooler, more-dense liquid or gas sinks. This causes convection within the liquid or gas as there is movement of particles within the material. Hence, heat input into the liquid or gas near the heat source is transferred to other places by the warm, less-dense fluid.
- The surface area exposed and the temperature difference between the fluid and the second material providing the heat.

Section 2.4 Radiation

KEY QUESTIONS SOLUTIONS

- 1
 - a The light can be partially reflected, partially transmitted and partially absorbed.
 - b Absorption of light is associated with temperature increase.
- 2 The higher the temperature of the object, the **higher** the frequency and the **shorter** the wavelength of the radiation emitted. For example, if a particular object emits radiation in the visible range, a cooler one could emit light in the **infrared** range of the electromagnetic spectrum.
- 3 E. The rate of emission or absorption will depend upon:
 - the temperature of the object and of the surrounding environment
 - the surface area of the object
 - the wavelength of the radiation
 - the surface characteristics of the object (e.g. its colour, and whether it is shiny or dull).
- 4 Conduction and convection require the presence of particles to transfer heat. Heat transfer by radiation can occur in a vacuum as the movement of particles is not required.
- 5 The person will have a higher temperature than their surroundings, and so will emit stronger infrared radiation than their surroundings. The infrared radiation is detected by the thermal imaging technology. The human eye cannot always distinguish a person from their surroundings, especially if they are under cover or if their clothes blend with the background.
- 6
 - a The matte black beaker cools faster than the others because matte black objects emit radiant energy faster than shiny, white surfaces.
 - b The gloss white surface will cool the slowest due to its light colour and shiny finish.
- 7 Heat sinks are made of dark-coloured metals that radiate heat energy strongly and keep the computer cool.

CHAPTER 2 REVIEW

- 1
 - a $\eta = \frac{\text{output}}{\text{input}} \times 100\%$
 Incandescent:
 $2 = \frac{\text{output}}{1000} \times 100\%$
 $\text{output} = \frac{2}{100} \times 1000$
 $\text{output} = 20\text{J}$
 - b LED
 $15 = \frac{\text{output}}{1000} \times 100\%$
 $\text{output} = \frac{15}{100} \times 1000$
 $\text{output} = 150\text{J}$
- 2 Running costs of the incandescent lights are 7.5 times that of the LED lights.
 $\text{ratio} = \frac{15\%}{2\%} = 7.5$
- 3 $U = W + Q$
 $= 520 + 1850$
 $= 2370\text{J}$
- 4 $Q = mc \Delta T$
 $T = \frac{Q}{mc}$
 $= \frac{2370}{0.200 \times 4180}$
 $= 2.83^\circ\text{C}$
 $T_{\text{final}} = T_{\text{initial}} + \Delta T$
 $= 20.0 + 2.83$
 $= 22.8^\circ\text{C}$

- 5 $\Delta Q = mc\Delta T$
 $= 0.200 \times 4180 \times (21.50 - 20.0)$
 $= 1254 \text{ J}$
- 6 $\eta = \frac{\text{output}}{\text{input}} \times 100\%$
 $= \frac{1254}{2370} \times 100$
 $= 52.9\%$
- 7 C. This will always be from the hottest to the coldest, i.e. from the object with the highest average internal kinetic energy.
- 8 **a** convection; **b** conduction
- 9 **a** The end of the poker that is not in the fire is warmed through conduction.
b You will feel the heat primarily through radiation.
c Heat escapes primarily due to conduction.
- 10 radiation
- 11 radiation and convection (primarily convection)
- 12 The Earth radiates an amount of energy into space equal to the amount it receives. This is affected by the composition of the atmosphere and the reflective index of the Earth and the atmosphere. Changes in either would lead to a change in the equilibrium position and a hotter or cooler Earth.
- 13 The function of the evacuated enclosure between the walls of a vacuum flask is to reduce heat losses due to conduction. (As seen in question 10, the silver coating on the walls reduces losses due to radiation.)
- 14 Expose both surfaces to a heater under the same temperature and environmental conditions. Measure the time each takes to heat to a particular temperature or measure the temperature of each surface after the same time. Thermal blankets are one real-world example.
- 15 Premature babies can lose a lot of moisture through their skin by evaporation. For a baby in a very warm environment, like an incubator at 37°C, there will be a large evaporative effect. A significant increase in evaporation occurs at incubator temperatures, and that evaporation of moisture from the baby will cool the baby dramatically. Thus an incubator must have not only a high temperature but also a high humidity. Other factors might include radiative energy loss, blood vessels being close to the skin surface and so there is less insulation than in an older baby.
- 16 Snow has a low thermal conductivity because it has many tiny air pockets trapped in its structure. Since this air-filled snow has a low thermal conductivity, the snow will not conduct much heat away from an object covered in it.
- 17 Both will be at the same temperature, matching that of their surroundings.
- 18 While paper is a better insulator and the can is a better conductor, the can will have a greater mass and hence take longer to heat up.
- 19 As cold water is denser than hot water, replacement water should enter at the bottom of the tank. Hot water should be drawn off at the top.
- 20 Air is a poorer conductor of heat than water. Hence, the rate of heat loss in air is less than the rate of heat loss in water. You transfer heat more quickly to the water and thus feel cold. Referring to Chapter 1, the specific heat capacity of water is higher than air so the water in contact with your body will heat up less quickly than the air in contact with your body. This also has the effect of increasing heat transfer away from your body.

Chapter 3 Particles in the nucleus

Section 3.1 Atoms, isotopes and radioisotopes

Worked example: Try yourself 3.1.1

WORKING WITH ATOMIC NOTATION

How many protons, neutrons, nucleons and electrons are there in ${}_{92}^{252}\text{U}$?	
Thinking	Working
The lower number is the atomic number, Z . This is the number of protons.	atomic number, $Z = 92$ This nuclide has 92 protons.
The upper number is the mass number, A . This indicates the number of particles in the nucleus, i.e. the number of nucleons.	mass number, $A = 252$ This nuclide has 252 nucleons.
The number of neutrons, N , is the difference between the mass number, A (the number of nucleons), and the atomic number, Z (the number of protons).	$N = A - Z$ $= 252 - 92$ $= 160$ This nuclide has 160 neutrons.
In an electrically neutral atom the number of protons = the number of electrons.	The nuclide has 92 protons, so the atom will have 92 electrons in the electron cloud.

Worked example: Try yourself 3.1.2

WORKING WITH ISOTOPES

Consider the isotope of thorium, ${}_{90}^{230}\text{Th}$. Work out the number of protons, nucleons and neutrons in this isotope.	
Thinking	Working
The lower number is the atomic number, Z . This is the number of protons.	atomic number, $Z = 90$ This nuclide has 90 protons.
The upper number is the mass number, A . This is the number of particles in the nucleus, i.e. the number of nucleons.	mass number, $A = 230$ This nuclide has 230 nucleons.
Subtract the atomic number, Z , from the mass number, A , to find the number of neutrons, N .	$N = A - Z$ $= 230 - 90$ $= 140$ This isotope has 140 neutrons.

Section 3.1 Review

KEY QUESTIONS SOLUTIONS

- nucleons
- 79 protons and 118 neutrons ($N = A - Z = 197 - 79 = 118$)
- 235
- Chlorine-35 has 17 protons, 18 neutrons ($35 - 17$) and 35 nucleons.
 - Plutonium-239 has 94 protons, 145 neutrons ($239 - 94$) and 239 nucleons.

- 5 B and D. Carbon has 6 protons, so ^{13}C has 7 neutrons ($13 - 6$). Nitrogen has 7 protons, so ^{14}N has 7 neutrons ($14 - 7$).
- 6 The number of electrons in a neutral atom is the same as the number of protons, which is given by the atomic number.
- 7 Isotopes are atoms with the same number of protons but different numbers of neutrons.
- 8
 - a Their atomic numbers are the same as they are both krypton. Their mass numbers (84 and 89) are different as they are isotopes and have different numbers of neutrons.
 - b There would be no difference in their chemical interactions with other atoms.
- 9 A radioisotope is an unstable isotope. At some time, it will spontaneously eject radiation in the form of alpha particles, beta particles or gamma rays from the nucleus.
- 10 Yes, a natural isotope can be radioactive. For example, uranium is naturally occurring and every isotope of uranium is radioactive.
- 11 Since the nuclear strong force acts only over a short range, for larger nuclei more neutrons, compared to protons, are needed to balance the long-range electrostatic force.

Section 3.2 Radioactivity

Worked example: Try yourself 3.2.1

ALPHA DECAY

A radium-224 nucleus is known to decay to a new element through the emission of an alpha particle. Determine the new element, write its symbol and write the decay equation.	
Thinking	Working
From the periodic table, radium-224 has 88 protons. Therefore its atomic number, Z , is 88 and its mass number, A , is 224.	It can be written $^{224}_{88}\text{Ra}$.
The initial nucleus is $^{224}_{88}\text{Ra}$ and is written on the left-hand side of the equation. The unknown nucleus is a result of alpha decay (^4_2He) and is written on the right-hand side along with the alpha particle.	$^{224}_{88}\text{Ra} \rightarrow ^A_Z\text{X} + \alpha$ or $^{224}_{88}\text{Ra} \rightarrow ^A_Z\text{X} + ^4_2\text{He}$
Charge must be conserved, so the total number of protons, Z , must be the same.	$88 = Z + 2$ $Z = 86$
The number of protons and neutrons, A , must also be the same.	$224 = A + 4$ $A = 220$
For the new element, $Z = 86$. From the periodic table, this is radon.	$^{220}_{86}\text{Rn}$
The decay equation can now be written.	$^{224}_{88}\text{Ra} \rightarrow ^{220}_{86}\text{Rn} + ^4_2\text{He}$

Worked example: Try yourself 3.2.2

BETA-MINUS DECAY

An astatine-219 nucleus is known to decay to a new element through the emission of a beta-minus particle. Determine the new element, write its symbol and write the decay equation.	
Thinking	Working
From the periodic table, astatine-219 has 85 protons. Therefore its atomic number, Z , is 85 and its mass number, A , is 219.	It can be written ${}_{85}^{219}\text{At}$.
The initial nucleus is ${}_{85}^{219}\text{At}$ and is written on the left-hand side of the equation. The unknown nucleus is a result of beta-minus decay and is written on the right-hand side along with the beta-minus particle and an antineutrino.	${}_{85}^{219}\text{At} \rightarrow {}_Z^AX + {}_{-1}^0\beta + \bar{\nu}$.
Charge must be conserved, so the total number of protons, Z , must be the same.	$85 = Z - 1$ $Z = 86$.
The number of protons and neutrons, A , must also be the same.	$219 = A + 0$ $A = 219$
For the new element, $Z = 86$. From the periodic table, this is radon.	${}_{86}^{219}\text{Rn}$
The decay equation can now be written.	${}_{85}^{219}\text{At} \rightarrow {}_{86}^{219}\text{Rn} + {}_{-1}^0\beta + \bar{\nu}$

Worked example: Try yourself 3.2.3

RADIOACTIVE DECAY 1

After beta-minus decay from boron to carbon-12, the carbon-12 atom is in an excited state and decays further to a more stable form of carbon-12. The equation is ${}_{6}^{12}\text{C}^* \rightarrow {}_{6}^{12}\text{C} + X$. Determine the atomic and mass numbers for X and identify the type of radiation being emitted.	
Thinking	Working
Balance the mass numbers.	The mass numbers of 12 are already balanced, so the mass number of X is zero.
Balance the atomic numbers.	The atomic numbers of 6 are already balanced, so the atomic number of X is zero
X has an atomic number of zero and a mass number of zero.	X is a gamma ray, γ .

Worked example: Try yourself 3.2.4

RADIOACTIVE DECAY 2

Polonium-218 decays by emitting an alpha particle and a gamma ray. The nuclear equation is: ${}_{84}^{218}\text{Po} \rightarrow X + {}_2^4\text{He} + \gamma$. Determine the atomic and mass numbers for X , then use the periodic table to identify the element.	
Thinking	Working
Balance the mass numbers.	$218 = A + 4$ mass number = 214
Balance the atomic numbers.	$84 = Z + 2$ $Z = 84 - 2 = 82$
Use the periodic table to look up element 82.	Element 82 is lead.

Section 3.2 Review

KEY QUESTIONS SOLUTIONS

- 1 mass number of X is $218 - 214 = 4$
atomic number of X is $86 - 84 = 2$
 X is an alpha particle.
- 2 mass number of Y is $214 - 214 = 0$
atomic number of Y is $82 - 83 = -1$
 Y is a beta-minus particle.
- 3 beta-plus
- 4 A positron is a positively charged electron.
- 5 Alpha is a helium nucleus. Beta is a positively or negatively charged electron ejected from the nucleus. Gamma is electromagnetic radiation.
- 6 40, 42, 43, 44, 46, 48
- 7 Alpha, beta and gamma radiation all originate from the nucleus of an atom.
- 8 **a** X : atomic number = $92 - 2 = 90$, mass number = $235 - 4 = 231$, X is thorium
b Y : atomic number = $88 + 1 = 89$, mass number = $228 + 0 = 228$, Y is actinium
- 9 **a** Nitrogen-14 has 7 protons and 7 neutrons.
b A neutron has changed into a proton, an electron and an antineutrino.
- 10 **a** atomic number = $20 - 21 = -1$, mass number = $45 - 45 = 0$, beta-minus particle
b atomic number = $70 - 68 = 2$, mass number = $150 - 146 = 4$, alpha particle

Section 3.3 Properties of alpha, beta and gamma radiation

Section 3.3 Review

KEY QUESTIONS SOLUTIONS

- 1 **a** gamma
b beta minus
c alpha
d beta
e gamma
- 2 gamma
- 3 beta
- 4 gamma
- 5 beta and gamma
- 6 **a** nucleus
b nucleus
c nucleus
- 7 gamma, beta, alpha
- 8 Alpha particles travel through air at a relatively low speed and have a double positive charge, which means they readily ionise the air. Their charge, ionising ability and their relatively slow speed make them very easy to stop. This means that they have a very poor penetrating ability.
- 9 The wire should be a beta emitter, since the irradiation needs to be confined to a relatively small area. Alpha radiation does not have sufficient penetrating power, while gamma radiation would irradiate adjacent healthy cells.
- 10 Alpha particles will all be stopped by the metal sheet. Gamma rays will all penetrate the metal sheet. Differences in the thickness of the metal sheet will not affect the count rates of these two.
Some beta particles will pass through thin metal, so for a set metal thickness there is a set rate of beta particles that should make it to the other side.

Section 3.4 Half-life and decay series

Worked example: Try yourself 3.4.1

HALF-LIFE

A sample of the radioisotope sodium-24 contains 4.0×10^{10} nuclei. The half-life of sodium-24 is 15 hours. How many sodium-24 atoms will remain in the sample after 150 hours?	
Thinking	Working
Calculate how many half-lives 150 hours corresponds to.	$n = \frac{150}{15}$ $= 10 \text{ half-lives}$
Substitute $N_0 = 4.0 \times 10^{10}$ and $n = 10$ into the equation. Calculate the number of nuclei remaining.	$N = N_0 \left(\frac{1}{2}\right)^n$ $= 4.0 \times 10^{10} \times \left(\frac{1}{2}\right)^{10}$ $= 3.9 \times 10^7 \text{ nuclei}$

Worked example: Try yourself 3.4.2

HALF-LIFE AND ACTIVITY

A sample of strontium-90 has an initial activity of 4000 Bq. Calculate its activity after 6 months using Table 3.4.1.	
Thinking	Working
Determine the half-life of strontium-90.	$t_{1/2} = 28.8 \text{ years}$
Calculate how many half-lives 6 months corresponds to. Convert the half-life into months first.	$t_{1/2} = 28.8 \times 12 \text{ months}$ $= 345.6 \text{ months}$ $n = \frac{6}{345.6}$ $= 0.0174 \text{ half-lives}$
Substitute the initial activity, $A_0 = 4000$, and the number of half-lives, $n = 0.0174$, into the equation. Calculate the final activity.	$A = A_0 \left(\frac{1}{2}\right)^n$ $= 4000 \times \left(\frac{1}{2}\right)^{0.0174}$ $= 4000 \times 0.988$ $= 3950 \text{ Bq}$ <p>Even after 6 months the activity has not significantly changed.</p>

Section 3.4 Review

KEY QUESTIONS SOLUTIONS

- The activity is the count rate or the number of decays each second.
- $$N = 8.0 \times 10^{10} \times \left(\frac{1}{2}\right)^1$$

$$= 4.0 \times 10^{10}$$
- $$n = \frac{24}{8}$$

$$n = 3$$

$$N = 2.4 \times 10^{12} \times \left(\frac{1}{2}\right)^3$$

$$= 3.0 \times 10^{11}$$

- 4 a Halve successively from a starting number, e.g. 800, until 0.1% of 800 (0.8) is reached:
 $800 \rightarrow 400 \rightarrow 200 \rightarrow 100 \rightarrow 50 \rightarrow 25 \rightarrow 12.5 \rightarrow 6.25 \rightarrow 3.125 \rightarrow 1.56 \rightarrow 0.78$.
 This takes 10 halvings
 or
 $0.1\% = 0.001$
 $\left(\frac{1}{2}\right)^n = 0.001$
 take logs of both sides
 $n \log\left(\frac{1}{2}\right) = \log 0.001$
 $-0.3n = -3$
 $n = 10$
 It will take 10 half-lives to fall below 0.1%.
- b 10 half-lives must pass = $10 \times 24\,000 = 240\,000$ years
- 5 The percentage chance any atom has of decaying in a period of time equal to its half-life is always 50%.
- 6 number of half-lives = 4
 $12 = N \times \left(\frac{1}{2}\right)^4$
 $12 = N \times 0.0625$
 $N = 12 \div 0.0625 = 192$
 so 192 μg must be produced.
- 7 $6000 \rightarrow 3000 \rightarrow 1500 \rightarrow 750 \rightarrow 375$
 So 4 half-lives have passed:
 $60 \div 4 = 15$
 So the half-life of the radioisotope is 15 minutes.
- 8 a time to fall from 800 \rightarrow 400 = 10 minutes or from 400 \rightarrow 200 = 10 minutes
 b 40 minutes = 4 half-lives; $A = 800 \times \left(\frac{1}{2}\right)^4 = 50$ Bq
- 9 Lead-210 undergoes beta decay. Its half-life is 20 years.
- 10 Starting from uranium-234, seven alpha and four beta-minus decays have occurred.

Section 3.5 Radiation dose and its effect on humans

Worked example: Try yourself 3.5.1

ABSORBED DOSE

A cancer tumour is exposed to 0.50J of radiation energy. The absorbed dose is 3Gy. Calculate the mass of the tumour. Assume that all of the radiation is absorbed by the tissue.	
Thinking	Working
Rearrange the equation $AD = \frac{E}{m}$.	$m = \frac{E}{AD}$
Substitute the values into the equation and solve.	$m = \frac{3}{0.5}$ mass of tissue = 6 kg

Worked example: Try yourself 3.5.2

DOSE EQUIVALENT

Calculate the dose equivalent (in mSv) from various radioactive sources if the absorbed dose is 1.25 mGy.	
Thinking	Working
The quality factor for each type of radiation can be found in Table 3.5.1. $1 \text{ mGy} = 1 \times 10^{-3} \text{ Gy}$	QF (alpha particles) = 20 QF (beta particles) = 1 QF (gamma rays) = 1

a Calculate the dose equivalent (in mSv) from a radiation source if the absorbed dose is 1.25 mGy and the source is emitting alpha particles.

Thinking

The dose equivalent, $DE = AD \times QF$.
Convert the answer to mSv using
 $1 \text{ mSv} = 1 \times 10^{-3} \text{ Sv}$.

Working

$$\begin{aligned} DE(\alpha) &= 1.25 \times 10^{-3} \times 20 \\ &= 0.025 \text{ Sv} \\ &= 25 \text{ mSv} \end{aligned}$$

b Calculate the dose equivalent (in mSv) from a radiation source if the absorbed dose is 1.25 mGy and the source is emitting beta particles.

Thinking

The dose equivalent, $DE = AD \times QF$.
Convert the answer to mSv.

Working

$$\begin{aligned} DE(\beta) &= 1.25 \times 10^{-3} \times 1 \\ &= 1.25 \times 10^{-3} \\ &= 1.25 \text{ mSv} \end{aligned}$$

c Calculate the dose equivalent (in mSv) from a radiation source if the absorbed dose is 1.25 mGy and the source is emitting gamma rays.

Thinking

The dose equivalent, $DE = AD \times QF$.
Convert the answer to mSv.

Working

$$\begin{aligned} DE(\gamma) &= 1.25 \times 10^{-3} \times 1 \\ &= 1.25 \times 10^{-3} \\ &= 1.25 \text{ mSv} \end{aligned}$$

Worked example: Try yourself 3.5.3

TREATING TUMOURS

A 25 g cancer tumour absorbs $5.0 \times 10^{-3} \text{ J}$ of energy from an applied radiation source. Calculate the dose equivalent if the source is an alpha emitter, using information from Table 3.5.1.

Thinking

Convert mass from grams to kg.

Working

$$\begin{aligned} m &= \frac{25}{1000} \\ &= 0.025 \text{ kg} \end{aligned}$$

Calculate the absorbed dose (AD) using the energy and the mass.

$$\begin{aligned} AD &= \frac{E}{m} \\ &= \frac{5 \times 10^{-3}}{0.025} \\ &= 0.2 \text{ Gy} \end{aligned}$$

Calculate the dose equivalent (DE) using the quality factor for alpha particles of 20.

$$\begin{aligned} DE &= AD \times QF \\ &= 0.2 \times 20 \\ &= 4 \text{ Sv} \end{aligned}$$

Section 3.5 Review

KEY QUESTIONS SOLUTIONS

- A. The unit, Gy, indicates that this is the absorbed dose. 1 Gy of alpha radiation will cause 20 times the damage to human tissue than beta or gamma radiation.
- D. The unit, Sv, indicates that this is the dose equivalent. $250 \mu\text{Sv}$ of any of the types of radiation stated will cause the same amount of damage.

- 3 a Absorbed dose, $AD = 200 \mu\text{Gy} = 200 \times 10^{-6} \text{Gy}$
 Dose equivalent, $DE = AD \times QF = 200 \times 10^{-6} \times 1 = 200 \times 10^{-6} \text{Sv} = 200 \mu\text{Sv}$
- b Absorbed dose, $AD = \frac{E}{m}$
 Absorbed energy, $E = AD \times m = 200 \times 10^{-6} \times 80 = 0.016 \text{ J}$
- 4 The amounts given are in Gy so they are the absorbed dose. Use $DE = AD \times QF$ to calculate the equivalent dose for each:
- A** $DE = 250 \times 10^{-6} \times 1 = 250 \times 10^{-6} \text{Sv} = 250 \mu\text{Sv}$
B $DE = 20 \times 10^{-6} \times 20 = 400 \times 10^{-6} \text{Sv} = 400 \mu\text{Sv}$
C $DE = 50 \times 10^{-6} \times 1 = 50 \times 10^{-6} \text{Sv} = 50 \mu\text{Sv}$
D $DE = 30 \times 10^{-6} \times 10 = 300 \times 10^{-6} \text{Sv} = 300 \mu\text{Sv}$
- The highest dose equivalent is $400 \mu\text{Sv}$, which comes from $20 \mu\text{Gy}$ of alpha radiation (B), followed by D, A, and C.
- 5 a The radiation doses are given in μSv so they are dose equivalents (DE).
 Convert normal background annual dose to μSv .
 $2 \text{ mSv} = 2 \times 10^3 \mu\text{Sv} = 2000 \mu\text{Sv}$
 Number of days = $\frac{2000}{1000} = 2 \text{ days}$
- b Number of days in space = 879 days
 Total radiation = $1000 \mu\text{Sv per day} \times \text{number of days}$
 $= 1000 \times 879$
 $= 879\,000 \mu\text{Sv} = 879 \text{ mSv}$
- This equates to 366 mSv per year . This is high but still slightly below the threshold for radiation sickness.
- 6 Number of hours = $\frac{\text{total absorbed dose}}{\text{absorbed dose per hour}}$
 $= \frac{36}{0.40}$
 $= 90 \text{ hours}$
- 7 D. A gamma emitter is needed for the camera, plus it minimises damage to the surrounding tissue as its ability to ionise tissue is low. A short half-life is needed to reduce patient exposure to radiation.

CHAPTER 3 REVIEW

- 1 20 protons and 25 neutrons ($45 - 20$)
- 2 Cobalt-60 has 27 protons, 33 neutrons ($60 - 27$) and 60 nucleons.
- 3 The atomic and mass numbers of X are both 0, so X is a gamma ray.
- 4 Potassium has $48 - 19 = 29$ neutrons. Figure 3.1.10 shows a minus sign so it emits a beta-minus particle.
- 5 a beta minus
 b proton
 c alpha
 d neutron
 e gamma
 f beta plus (positron)
- 6 atomic number = $5 - 2 = 3$, mass number = $11 - 4 = 7$, so X is lithium
- 7 a atomic number = $(7 + 2) - 8 = 1$, mass number = $(14 + 4) - 17 = 1$, so X is a proton
 b atomic number = $(12 + 1) - 13 = 0$, mass number = $(27 + 1) - 27 = 1$, so Y is a neutron
- 8 a $208 = x + 0 \rightarrow x = 208$
 $81 = y - 1 \rightarrow y = 82$
 b $180 = x + 4 \rightarrow x = 176$
 $80 = y + 2 \rightarrow y = 78$
- 9 $18 = a + 0 \rightarrow a = 18$
 $10 = b + 1 \rightarrow b = 9$
 $18 = c + 0 \rightarrow c = 18$
 $9 = d + 1 \rightarrow d = 8$
- X has atomic number 9, which is fluorine, F.
 Y has atomic number 8, which is oxygen, O.

- 10** atomic number = 12, mass number = $7 - 1 = 6$, X is carbon-12
- 11** Electromagnetic forces are balanced by the strong nuclear force acting between all nucleons in close proximity.
- 12** **a** gamma
b gamma
- 13** All types of ionising radiation, including alpha, beta, and gamma.
- 14** The bombarding electrons will be strongly repelled by the electron clouds of the atoms as they are all negatively charged. The small mass of the bombarding electrons also makes them relatively easy to repel compared to, for example, a proton.
- 15** 1 half-life has passed so

$$N = 6.0 \times 10^{14} \times \left(\frac{1}{2}\right)^1 = 3.0 \times 10^{14}$$
- 16** 3 half-lives have passed so

$$N = 5.6 \times 10^{15} \times \left(\frac{1}{2}\right)^3 = 7.0 \times 10^{14}$$
- 17** To have a shorter half-life, the nuclei are decaying at a faster rate, so uranium-235 has a greater activity.
- 18** **a** After one half-life the activity halves to 2 MBq.
b 6 hours
c 18 hours pass = 3 half-lives

$$4.0 \times 10^6 \times \left(\frac{1}{2}\right)^3 = 5.0 \times 10^5 \text{ Bq}$$
- 19** 2 half-lives pass

$$N = 6.0 \times 10^{10} \times \left(\frac{1}{2}\right)^2 = 1.5 \times 10^{10}$$
- 20** The long half-life means that the source will not need to be replaced for many years. The gamma rays have a strong penetrating power so they are able to penetrate the skull and reach the tumour site.
- 21** $DE = AD \times QF = 300 \times 10^{-3} \times 1 = 300 \times 10^{-3} = 300 \text{ mSv}$
- 22** **a** $AD = \frac{E}{m}$ so $E = AD \times m = 5 \times 75 = 375 \text{ J}$
b $DE = AD \times QF = 5 \times 1 = 5 \text{ Sv}$
- 23** **a** Worker works for $45 \times 5 = 225$ days
 Number of X-ray photographs = $225 \times 10 = 2250$ X-rays
 Dose per X-ray = $\frac{7900}{2250} = 3.51 \mu\text{Sv}$
- b** Normal background is 1.5 mSv in Australia = $1500 \mu\text{Sv}$
 Number of times greater = $\frac{7900}{1500} = 5.3$ times

Chapter 4 Fission and fusion

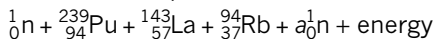
Section 4.1 Nuclear fission and energy

Worked example: Try yourself 4.1.1

FISSION

Plutonium-239 is a fissile material. When a plutonium-239 nucleus is struck by and absorbs a neutron, it can split in many different ways. Consider the example of a nucleus that splits into lanthanum-143 and rubidium-94 and releases some neutrons.

The nuclear equation for this is:



a How many neutrons are released during this fission process, i.e. what is the value of a ?	
Thinking	Working
Analyse the mass numbers (A).	$1 + 239 = 143 + 94 + (a \times 1)$ $a = (1 + 239) - (143 + 94)$ $= 3$ 3 neutrons are released during this fission.
b During this single fission reaction, there is a loss of mass (a mass defect) of 4.58×10^{-28} kg. Calculate the amount of energy that is released during fission of a single plutonium-239 nucleus. Give your answer in both MeV and joules to two significant figures.	
Thinking	Working
The energy released during the fission of this plutonium nucleus can be found by using $\Delta E = \Delta mc^2$.	$\Delta E = \Delta mc^2$ $= (4.58 \times 10^{-28}) \times (3.00 \times 10^8)^2$ $= 4.12 \times 10^{-11} \text{ J}$
To convert J into eV, divide by 1.6×10^{-19} . Remember that $1 \text{ MeV} = 10^6 \text{ eV}$.	$\Delta E = \frac{4.12 \times 10^{-11}}{1.6 \times 10^{-19}}$ $= 2.58 \times 10^8 \text{ eV}$ $= 258 \text{ MeV}$
c The combined mass of the plutonium nucleus and bombarding neutron is 2.86×10^{-25} kg. What percentage of this initial mass is converted into the energy produced during the fission process?	
Thinking	Working
Use the relationship percentage of initial mass converted into energy $= \frac{\text{mass defect}}{\text{initial mass}} \times \frac{100}{1}$	percentage of initial mass converted into energy $= \frac{\text{mass defect}}{\text{initial mass}} \times \frac{100}{1}$ $= \frac{4.58 \times 10^{-28}}{2.86 \times 10^{-25}} \times \frac{100}{1}$ $= 0.16\%$

Section 4.1 Review

KEY QUESTIONS SOLUTIONS

- The strong nuclear force is a force of attraction that acts between every nucleon but only over relatively short distances. This force acts like a nuclear cement.
- The decay products of the nuclear fission process comprise many different, often highly radioactive isotopes. This is what makes up the waste.
- As the neutron is neutral it will only experience attractive forces from other nucleons due to the strong nuclear force.
- $5.0 \times 10^6 \times 1.6 \times 10^{-19} = 8.0 \times 10^{-13} \text{ J}$

- 5 $\frac{6.0 \times 10^{-15}}{1.6 \times 10^{-19}} = 3.8 \times 10^4 \text{ eV}$
- 6 Fissile—uranium-235 and plutonium-239
Non-fissile—uranium-238 and cobalt-60
- 7 Balance the mass numbers: $1 + 235 = 148 + 85 + x$
 $x = 3$
- 8 a $\Delta E = \Delta mc^2$
 $= (2.12 \times 10^{-28}) \times (3.00 \times 10^8)^2$
 $= 1.91 \times 10^{-11} \text{ J}$
b $1 \text{ J} = 1.6 \times 10^{-19} \text{ eV}$
Energy in eV $= \frac{1.9 \times 10^{-11}}{1.6 \times 10^{-19}}$
 $= 1.19 \times 10^8 \text{ eV}$
- 9 $\Delta E = \Delta mc^2$
 $= 3.48 \times 10^{-28} \times (3 \times 10^8)^2$
 $= 3.13 \times 10^{-11} \text{ J}$
 $1 \text{ J} = 1.6 \times 10^{-19} \text{ eV}$
Energy in eV $= \frac{3.13 \times 10^{-11}}{1.6 \times 10^{-19}}$
 $= 1.96 \times 10^8 \text{ eV}$
- 10 Energy in J $= 1.33 \times 10^6 \times 1.6 \times 10^{-19} = 2.128 \times 10^{-13} \text{ J}$
 $m = \frac{E}{c^2}$
 $= \frac{2.128 \times 10^{-13}}{(3 \times 10^8)^2}$
 $= 2.36 \times 10^{-30} \text{ kg}$
- 11 Balance the mass numbers:
 $1 + x = 130 + 106 + 4$
 $x = 239$
Balance the atomic numbers:
 $0 + 94 = 54 + y + 0$
 $y = 40$

Section 4.2 Chain reactions and nuclear reactors

Section 4.2 Review

KEY QUESTIONS SOLUTIONS

- 1 B. Uranium-235 is highly fissionable with slow neutrons. The reaction produces two daughter nuclei, more neutrons and energy.
- 2 There not a high enough concentration of fissile uranium-235 to sustain a chain reaction.
- 3 B. This is the concentration needed to sustain a chain reaction in the reactor core.
- 4 The moderator slows neutrons, which allows them to induce fission in the nuclear fuel.
- 5 Control rods absorb neutrons and maintain a controlled chain reaction.

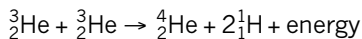
- 6 The mass of material must exceed the critical mass and it must have the correct shape to sustain a chain reaction.
- 7 As a result of the flat shape a high proportion of the neutrons emitted in the fission reaction will escape.
- 8 The lead nucleus is too heavy so the incident neutron will keep most of its energy after collision. It will not have slowed down sufficiently to be captured by a fissile nucleus.
- 9 Parts a and c would sustain a chain reaction; part b would not be able to sustain a chain reaction.
- 10 a A fast neutron is unlikely to be captured by a nucleus.
b A slow neutron is likely to be absorbed by the nucleus and cause fission.
- 11 a The uranium-238 will transmute to plutonium-239.
b Plutonium-239 is highly radioactive, with a half-life of 24 000 years, so will need to be stored for a long time.
- 12 Only one neutron is needed to sustain a chain reaction, leaving the remaining neutrons to breed more plutonium.
- 13 a ${}_{92}^{238}\text{U} + {}_0^1\text{n} \rightarrow {}_{92}^{239}\text{U}$
 ${}_{92}^{239}\text{U} \rightarrow {}_{93}^{239}\text{Np} + {}_0^0\text{e}$
 ${}_{93}^{239}\text{Np} \rightarrow {}_{94}^{239}\text{Pu} + {}_0^0\text{e}$
 b ${}_{94}^{239}\text{Pu} + {}_0^1\text{n} \rightarrow {}_{54}^{134}\text{Xe} + {}_{40}^{103}\text{Zr} + 3{}_0^1\text{n}$
- 14 Over time the number of fissile nuclei in the fuel rods becomes depleted, resulting in a reduced number of fission reactions and hence fewer mobile neutrons in the core. In order to maintain a chain reaction the control rods must be gradually withdrawn over time.

Section 4.3 Nuclear fusion

Worked example: Try yourself 4.3.1

FUSION

One of the possible nuclear fusion reactions in a star involves the fusion of two helium-3 nuclei to produce a helium-4 nucleus, two protons and energy according to the equation below. Calculate the energy, in joules and MeV, released in this reaction. Use the following data in your calculations: mass of helium-3 nucleus = 3.014 932 u, mass of helium-4 nucleus = 4.001 505 u and mass of a proton = 1.007 276 u.



Thinking	Working
Determine the mass of the reactants.	$\text{mass of reactants} = 2 \times \text{mass of helium-3}$ $= 2 \times 3.014\,932\text{ u}$ $= 6.029\,864\text{ u}$
Determine the mass of the products.	$\text{mass of products} = \text{mass of helium-4} + 2 \times \text{mass of proton}$ $= 4.001\,505 + 2 \times 1.007\,276$ $= 6.016\,067\text{ u}$
Determine the mass defect.	$\text{mass defect} = \text{mass of reactants} - \text{mass of products}$ $= 6.029\,864 - 6.016\,067$ $= 0.013\,797\text{ u}$
Determine the energy equivalent.	$= \text{mass defect} \times 931\text{ MeV}$ $= 0.013\,797 \times 931\text{ MeV}$ $= 12.85\text{ MeV}$
Convert to joules.	$= 12.85 \times 10^6 \times 1.60 \times 10^{-19}$ $= 2.06 \times 10^{-12}\text{ J}$

Worked example: Try yourself 4.3.2
BINDING ENERGY

Calculate the average binding energy per nucleon for the uranium-235 nucleus in MeV and joules. Use the following data in your calculations: mass of a uranium-235 nucleus = 234.993 462 u, mass of a proton = 1.007 276 u and mass of a neutron = 1.008 664 u.

Thinking	Working
Determine the total mass of the nucleons in a uranium-235 nucleus.	total mass = mass of 143 neutrons + mass of 92 protons = $143 \times 1.008\,664 + 92 \times 1.007\,276$ = 236.908 344 u
Determine the mass defect.	= mass of nucleons – actual mass of nucleus = $236.908\,344 - 234.993\,462$ = 1.914 882 u
Determine the binding energy in MeV.	= mass defect \times 931 MeV = $1.914\,882 \times 931$ = 1784 MeV
Determine the binding energy per nucleon in MeV.	= $\frac{1784}{235}$ = 7.59 MeV per nucleon
Determine the binding energy per nucleon in J.	= $7.59 \times 10^6 \times 1.60 \times 10^{-19}$ J = 1.21×10^{-12} J

Section 4.3 Review

KEY QUESTIONS SOLUTIONS

- 1 Fusion is the joining together of two small nuclei to form a larger nucleus. Fission is the splitting apart of one large nucleus into smaller fragments.
- 2 The mass of the products is less than the mass of the reactants. The mass difference is related to the energy released via $\Delta E = \Delta mc^2$.
- 3 The amount of energy released per nucleon during a single nuclear fission reaction is less than the amount for a single fusion reaction.

4 less than 1%

- 5 a Balance the mass numbers:

$$2 + 3 = a + 1$$

$$a = 4$$

Balance the atomic numbers:

$$1 + 1 = b + 0$$

$$b = 2$$

X is helium, He

- b $\Delta E = \Delta mc^2$

$$\Delta m = \frac{E}{c^2}$$

$$= \frac{33 \times 10^6 \times 1.6 \times 10^{-19}}{(3.0 \times 10^8)^2}$$

$$\Delta m = 5.9 \times 10^{-29} \text{ kg}$$

- 6 Electrostatic forces of repulsion act on the protons. If the protons are moving slowly they will not have enough energy to overcome the repulsive forces and they will not fuse together.
- 7 Electrostatic forces of repulsion act on the protons, but they are travelling fast enough to overcome these forces. The protons will get close enough for the strong nuclear force to take effect and they will fuse together. These protons have overcome the energy barrier.
- 8 a Balance the mass numbers:
 $4 + 1 + 1 - 3 = 3$
 Balance the atomic numbers:
 $2 + 1 + 1 - 2 = 2$
 Particle X is ${}^3_2\text{He}$
- b $\Delta E = 23 \times 10^6 \times 1.6 \times 10^{-19} = 3.7 \times 10^{-12} \text{ J}$
- c $\Delta E = \Delta mc^2$

$$\Delta m = \frac{E}{c^2}$$

$$= \frac{3.7 \times 10^{-12}}{(3 \times 10^8)^2}$$

$$= 4.1 \times 10^{-29} \text{ kg}$$
- 9 When two hydrogen-2 nuclei are fused together to form a helium-4 nucleus, the binding energy per nucleon increases and the nucleus becomes more stable.
- 10 The number of nucleons is conserved as there are five nucleons on each side of the reaction.

CHAPTER 4 REVIEW

- 1 A nuclide that is able to split in two when hit by a neutron is fissile.
- 2 No, only a few nuclides (e.g. uranium-235 and plutonium-239) are fissile.
- 3 The strong nuclear force causes the proton to be attracted to all other nucleons. It will also experience a smaller electrostatic force of repulsion between itself and other protons.
- 4 Neutrons are uncharged and are not repelled by the nucleus as alpha particles are.
- 5 a $\Delta E = \Delta mc^2$
 $= 3.48 \times 10^{-28} \times (3.0 \times 10^8)^2$
 $= 3.1 \times 10^{-11} \text{ J}$
- b $\Delta E = \frac{3.1 \times 10^{-11}}{1.6 \times 10^{-19}}$
 $= 1.96 \times 10^8$
 $= 196 \text{ MeV}$
- 6 $1 + x = 130 + 106 + 4 \times 1$
 $x = 239$
 $0 + 94 = 54 + y + 4 \times 0$
 $y = 40$
- 7 Balance the mass numbers:
 $1 + 235 = 127 + 102 + x$
 $x = 7$
- 8 The nuclei are all positively charged and so repel each other. They need a massively large amount of energy to overcome these forces and get close enough for the strong nuclear force to take effect. 100 million degrees provides the required energy for this to occur.
- 9 $\Delta E = \Delta mc^2$
 $= 4.99 \times 10^{-28} \times (3.0 \times 10^8)^2$
 $= 4.49 \times 10^{-11} \text{ J}$

- 10 a** The combined mass of the hydrogen and helium-3 nuclei is greater than the combined mass of the helium-4 nucleus, positron and neutrino.
- b** The energy has come from the lost mass (or mass defect) via $\Delta E = \Delta mc^2$.
- c** $21 \text{ MeV} = 21 \times 10^6 \times 1.6 \times 10^{-19} = 3.4 \times 10^{-12} \text{ J}$
- d** $\Delta E = \Delta mc^2$

$$\Delta m = \frac{E}{c^2}$$

$$= \frac{3.4 \times 10^{-12}}{(3 \times 10^8)^2}$$

$$= 3.8 \times 10^{-29} \text{ kg}$$
- 11** Fission reactors create a great deal more waste. Fusion releases more energy per nucleon than fission.
- 12** The binding energy per nucleon increases and the nucleus becomes more stable.
- 13** The higher the binding energy, the more stable the nucleus. This is because higher binding energy means that it takes more energy to completely separate particles in the nucleus. Iron therefore has the most stable nuclei of all the elements.
- 14** gamma rays
- 15** When uranium-238 absorbs neutrons and undergoes transmutation it produces plutonium-239 as one of the daughter nuclei.
- 16 a** The coolant transfers the heat from the reactor to the heat exchanger.
b The moderator slows down, or moderates, the speed of the neutrons.
c The control rods control the number of neutrons involved in the chain reaction.
- 17 a** ${}_1^1\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + \gamma$
- b** energy released = $20 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-12} \text{ J}$
- c** $m = \frac{E}{c^2}$

$$= \frac{3.2 \times 10^{-12}}{(3 \times 10^8)^2}$$

$$= 3.56 \times 10^{-29} \text{ kg}$$
- 18 a** ${}_1^1\text{H} + {}_1^1\text{H} \rightarrow {}_1^2\text{H} + {}_+1^0\text{e}$
 ${}_1^1\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + \gamma$
 ${}_2^3\text{He} + {}_2^3\text{He} \rightarrow {}_2^4\text{He} + 2{}_1^1\text{H}$
- b** Mass of reactants = $2 \times 3.01493 \times 1.66054 \times 10^{-27} = 1.00128 \times 10^{-26} \text{ kg}$
 Mass of products = $(4.00151 + 2 \times 1.00728) \times 1.66054 \times 10^{-27} = 9.98992 \times 10^{-27} \text{ kg}$
 Mass defect = $1.00128 \times 10^{-26} - 9.98992 \times 10^{-27} = 2.28751 \times 10^{-29} \text{ kg}$
 $\Delta E = \Delta mc^2 = 2.28751 \times 10^{-29} \times (3 \times 10^8)^2 = 2.05876 \times 10^{-12} \text{ J}$
 power = $\frac{\text{energy}}{\text{time}}$

$$= \frac{2.05876 \times 10^{-12}}{(24 \times 60 \times 60)}$$

$$= 2.38282 \times 10^{-17} \text{ W per reaction}$$

 Power from 100g = power per reaction \times number of reactions per 100g

$$= \frac{2.38282 \times 10^{-17} \times 0.1}{1.00128 \times 10^{-26}}$$

$$= 2.38 \times 10^8 \text{ W}$$

$$= 238 \text{ MW}$$

Chapter 5 Electrical physics

Section 5.1 Behaviour of charged particles

Worked example: Try yourself 5.1.1

AMOUNT OF CHARGE ON A GROUP OF ELECTRONS

Calculate the charge, in coulombs, carried by 4 million electrons.	
Thinking	Working
Express 4 million in scientific notation.	1 million = 10^6 4 million = 4×10^6
Calculate the charge, Q , in coulombs by multiplying the number of electrons by the charge on an electron ($-1.6 \times 10^{-19} \text{C}$).	$q = (4 \times 10^6) \times (-e)$ $= (4 \times 10^6) \times (-1.6 \times 10^{-19} \text{C})$ $= -6.4 \times 10^{-13} \text{C}$

Worked example: Try yourself 5.1.2

NUMBER OF ELECTRONS IN A GIVEN AMOUNT OF CHARGE

The net charge on an object is $-4.8 \mu\text{C}$ ($1 \mu\text{C} = 1 \text{ microcoulomb} = 10^{-6} \text{C}$). Calculate the number of extra electrons on the object.	
Thinking	Working
Express $-4.8 \mu\text{C}$ in scientific notation.	$q = -4.8 \mu\text{C}$ $= -4.8 \times 10^{-6} \text{C}$
Find the number of electrons by dividing the charge on the object by the charge on an electron. ($-1.6 \times 10^{-19} \text{C}$)	$n_e = \frac{q}{-e}$ $= \frac{-4.8 \times 10^{-6} \text{C}}{-1.6 \times 10^{-19} \text{C}}$ $= 3.0 \times 10^{13} \text{ electrons}$

Section 5.1 Review

KEY QUESTIONS SOLUTIONS

- They will attract as they will be oppositely charged.
- $n_e = \frac{-5.0 \text{C}}{-1.6 \times 10^{-19} \text{C}} = 3.1 \times 10^{19}$
- $q = 4.2 \times 10^{19} \times 1.6 \times 10^{-19} \text{C} = +6.7 \text{C}$
- Copper is a good conductor of electricity because its electrons are loosely held to their respective nuclei. This allows electrons to move freely through the material by 'jumping' from one atom to the next. Plastic is a good insulator. The plastic coating is used to insulate copper wiring to prevent charge leaving the circuit.

Section 5.2 Energy in electric circuits

Worked example: Try yourself 5.2.1

USING THE DEFINITION OF POTENTIAL DIFFERENCE

A car battery can provide 3600 C of charge at 12 V. How much electrical potential energy is stored in the battery?	
Thinking	Working
Recall the definition of potential difference.	$\Delta V = \frac{E}{q}$
Rearrange this to make energy the subject.	$E = \Delta Vq$
Substitute in the appropriate values and solve.	$E = 12 \times 3600$ $= 43\,200\text{ J}$ $= 4.3 \times 10^4\text{ J}$

Section 5.2 Review

KEY QUESTIONS SOLUTIONS

- A. When a conductor links two bodies between which there is a potential difference, charges will flow through the conductor until the potential difference is equal to zero.
- $\Delta V = \frac{E}{q}$
 - $\frac{40}{10} = 4\text{ V}$
 - $\frac{15}{10} = 1.5\text{ V}$
 - $\frac{20}{10} = 2\text{ V}$
- $\Delta V = \frac{E}{q}$
 $= \frac{100}{5}$
 $= 20\text{ V}$
- $E = \Delta Vq$
 $2 \times 10^3\text{ J} = q(12\text{ V})$
 $q = 167\text{ C}$
- heat and light
 - $E = \Delta Vq$
 $q = \frac{E}{V}$
 $= \frac{(3.6 \times 10^3)}{(240)}$
 $= 15\text{ C}$
- the gravitational potential energy of the water
- The voltmeter must always be in parallel with the light bulb, i.e. at M2 or M3.

Section 5.3 Electric current and circuits

Worked example: Try yourself 5.3.1

USING $I = \frac{q}{t}$

Calculate the number of electrons that flow past a particular point each second in a circuit that carries a current of 0.75 A.	
Thinking	Working
Rearrange the equation $I = \frac{q}{t}$ to make q the subject.	$I = \frac{q}{t}$ so $q = I \times t$
Calculate the amount of charge that flows past the point in question by substituting the values given.	$q = 0.75 \times 1$ $= 0.75 \text{ C}$
Find the number of electrons by dividing the charge by the charge on an electron ($1.6 \times 10^{-19} \text{ C}$).	$n_e = \frac{q}{q_e}$ $= \frac{0.75}{1.6 \times 10^{-19}}$ $= 4.69 \times 10^{18} \text{ electrons}$

Worked example: Try yourself 5.3.2

USING $E = \Delta VIt$

A potential difference of 12 V is used to generate a current of 1750 mA to heat water for 7.5 minutes. Calculate the energy transferred to the water in that time.	
Thinking	Working
Convert quantities to SI units.	$\frac{1750 \text{ mA}}{1000} = 1.75 \text{ A}$ $7.5 \text{ min} \times 60 \text{ s} = 450 \text{ s}$
Substitute values into the equation and calculate the amount of energy in joules.	$E = \Delta VIt$ $= 12 \times 1.750 \times 7.5 \times 60$ $= 9.45 \times 10^3 \text{ J}$

Worked example: Try yourself 5.3.3

USING $P = \Delta VI$

An appliance running on 120 V draws a current of 6 A. Calculate the power used by this appliance.	
Thinking	Working
Identify the relationship needed to solve the problem.	$P = \Delta VI$
Identify the required values from the question, substitute and calculate.	$P = 120 \times 6$ $= 720 \text{ W}$

Section 5.3 Review

KEY QUESTIONS SOLUTIONS

- 1 A continuous conducting loop (closed circuit) must be created from one terminal of a power supply to the other terminal.
- 2 Cell, light bulb, open switch, resistor and ammeter.
- 3 C. It's now known that charge carriers are electrons, which flow from the negative terminal to the positive terminal of the battery.
- 4 $I = \frac{q}{t}$ in coulombs and seconds
 - a 3 A
 - b 0.5 A
 - c 0.008 A
- 5 Use $q = It$, with I in amperes and t in seconds.
 - a 5 C
 - b 300 C
 - c 18 000 C
- 6
 - a $q = It = (5 \times 10^{-3}) \times (600) = 3 \text{ C}$
 - b $q = 200 \times 5 = 1000 \text{ C}$
 - c $q = (400 \times 10^{-3}) \times (3600) = 1440 \text{ C}$
- 7
 - a $q = n_e \times q_e$
 $= 10^{20} \times 1.6 \times 10^{-19} \text{ C}$
 $= 16 \text{ C}$
 - b $I = \frac{q}{t}$
 $= \frac{16}{4}$
 $= 4 \text{ A}$
- 8 3.2 C flow past a point in 10 seconds. Calculate:
 - a $n_e = \frac{q}{q_e}$
 $= \frac{3.2}{1.6 \times 10^{-19}}$
 $= 2 \times 10^{19}$ electrons
 - b $I = \frac{q}{t}$
 $= \frac{3.2}{10}$
 $= 0.32 \text{ A}$
- 9 B. Circuit B is the only circuit in which the current passes through both globes and the ammeter is in series with both globes when the switch is closed.
- 10
 - a $t = 5 \times 60 = 300 \text{ s}$
 $E = P \times t$
 $= 460 \times 300$
 $= 138\,000 \text{ J (or } 138 \text{ kJ)}$
 - b $I = \frac{P}{V}$
 $= \frac{460}{230}$
 $= 2 \text{ A}$

Section 5.4 Resistance

Worked example: Try yourself 5.4.1

USING OHM'S LAW TO CALCULATE RESISTANCE

An electric bar heater draws 10A of current when connected to a 240V power supply. Calculate the resistance of the element in the heater.	
Thinking	Working
Ohm's law is used to calculate resistance.	$\Delta V = IR$
Rearrange the equation to find R .	$R = \frac{V}{I}$
Substitute in the known values.	$R = \frac{240}{10}$ $= 24\Omega$

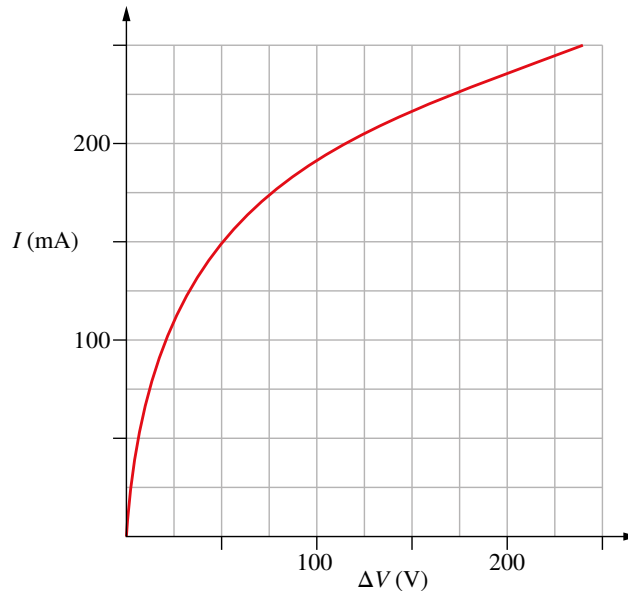
Worked example: Try yourself 5.4.2

USING OHM'S LAW TO CALCULATE RESISTANCE, CURRENT AND POTENTIAL DIFFERENCE

The table below shows measurements for the potential difference and corresponding current for an ohmic conductor.				
ΔV (V)	0	3	9	ΔV_2
I (A)	0	0.20	I_1	0.80
Determine the missing results, I_1 and V_2 .				
Thinking	Working			
Determine the factor by which potential difference has increased from the second column to the third column.	$\frac{9}{3} = 3$ The potential difference has tripled.			
Apply the same factor increase to the current in the second column, to determine the current in the third column (I_1).	$I_1 = 3 \times 0.20$ $= 0.6\text{A}$			
Determine the factor by which current has increased from the second column to the fourth column.	$\frac{0.80}{0.20} = 4$ The current has increased by a factor of 4.			
Apply the same factor increase to the potential difference in the second column, to determine the potential difference in the fourth column (ΔV_2).	$\Delta V_2 = 4 \times 3$ $= 12\text{V}$			

Worked example: Try yourself 5.4.3
CALCULATING RESISTANCE FOR A NON-OHMIC CONDUCTOR

A 240V, 60W incandescent light globe has the I - ΔV characteristics shown in the graph. Calculate the resistance of the light globe when the potential difference is 175V.



Thinking	Working
From the graph, determine the current at the required potential difference. Note that current is given in mA, so convert it to A.	At $\Delta V = 175\text{V}$, $I = 225\text{mA}$. Therefore $I = 0.225\text{A}$
Substitute these values into Ohm's law to find the resistance.	$R = \frac{V}{I}$ $= \frac{175}{0.225}$ $= 778\Omega$

Worked example: Try yourself 5.4.4
USING OHM'S LAW TO FIND CURRENT

The element of a bar heater has a resistance of 25Ω . Calculate the current (in mA) that will flow through this element if it is connected to a 240V supply.

Thinking	Working
Recall Ohm's law.	$\Delta V = IR$
Rearrange the equation to make I the subject.	$I = \frac{V}{R}$
Substitute in the values for this problem and solve.	$I = \frac{240}{25}$ $= 9.60\text{A}$
Convert the answer to the required units.	$I = 9.6\text{A}$ $= 9.60 \times 10^3\text{mA}$ $= 9600\text{mA}$

Worked example: Try yourself 5.4.5
USING OHM'S LAW TO FIND POTENTIAL DIFFERENCE

The globe of a torch has a resistance of $5.7\ \Omega$ when it draws 700 mA of current. Calculate the potential difference across the globe.	
Thinking	Working
Convert 700 mA to A.	$700 \times 10^{-3} = 0.7\ \text{A}$
Recall Ohm's law.	$\Delta V = IR$
Substitute in the known values and solve.	$\Delta V = 0.7 \times 5.7$ $= 3.99\ \text{V}$

Section 5.4 Review
KEY QUESTIONS SOLUTIONS

- 1 a A, B, C
b C, B, A

$$2 \quad R = \frac{V}{I}$$

$$= \frac{2}{0.25}$$

$$= 8\ \Omega$$

$$I = \frac{V}{R}$$

$$I_1 = \frac{3}{8}$$

$$= 0.375\ \text{A}$$

$$\Delta V = IR$$

$$\Delta V_2 = 0.60 \times 8$$

$$= 4.8\ \text{V}$$

- 3 a The wire is ohmic. This is because there is a proportional relationship between the voltage and the current, as shown by the linear nature of the I - ΔV graph, which means that the resistance is a constant.
b 3 A

$$c \quad R = \frac{V}{I}$$

$$= \frac{25}{10}$$

$$= 2.5\ \Omega$$

4 a $R = \frac{V}{I}$

$$= \frac{2.5}{3.5}$$

$$= 0.71\ \Omega$$

- b Since $V = IR$, if we double our potential difference, we would expect the current to double if the resistance of the ohmic resistor is constant. Therefore:

$$2V = (2 \times I)R$$

$$(2 \times 2.5) = (2 \times 3.5)R$$

$$5 = 7R$$

And we can see the value of R has remained constant.

- 5 They are both right. The resistance of the device is different for different voltages. Therefore the device is non-ohmic.

6 $R = \frac{5}{45 \times 10^{-3}} = 111.11\ \Omega$

$$I = \frac{8}{111.11} = 72\ \text{mA}$$

- 7 a $R = \frac{V}{I} = \frac{4}{2} = 2\ \Omega$
- b $I = \frac{10}{2} = 5\ \text{A}$
- 8 a $R = \frac{kL}{A}$
 $0.8 = \frac{kL}{A}$ then $1.6 = \frac{k(2L)}{A} = 1.6\ \Omega$
- b A wire of twice the diameter has four times the cross-sectional area.
 Then $R = \frac{0.8}{4} = 0.2\ \Omega$.
- 9 a It is non-ohmic, as the I - ΔV relationship is nonlinear.
- b From the graph, when $\Delta V = 10\ \text{V}$, $I = 0.5\ \text{A}$.
- c For $I = 1.0\ \text{A}$, $\Delta V = 15\ \text{V}$.
- d The resistance of the device at these voltages will be given by $R = \frac{V}{I}$.
- i For $10\ \text{V}$, $R = \frac{10}{0.5} = 20\ \Omega$.
- ii For $20\ \text{V}$, $R = \frac{20}{1.5} = 13.3\ \Omega$.

Section 5.5 Series and parallel circuits

Worked example: Try yourself 5.5.1

CALCULATING AN EQUIVALENT SERIES RESISTANCE

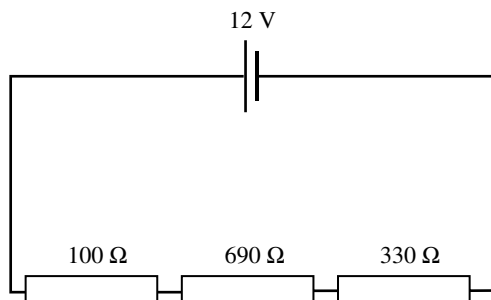
A string of Christmas lights consists of 20 light bulbs connected in series. Each bulb has a resistance of $8\ \Omega$. Calculate the equivalent series resistance of the Christmas lights.

Thinking	Working
Recall the formula for equivalent series resistance.	$R_T = R_1 + R_2 + \dots + R_n$
Substitute in the given values for resistance. As there are 20 equal globes, you can multiply $8\ \Omega$ by 20 globes.	$R_T = 20 \times 8$ $= 160\ \Omega$

Worked example: Try yourself 5.5.2

USING EQUIVALENT SERIES RESISTANCE FOR CIRCUIT ANALYSIS

Use an equivalent series resistance to calculate the current flowing in the series circuit below and the potential difference across each resistor.



Thinking	Working
Recall the formula for equivalent series resistance.	$R_T = R_1 + R_2 + R_3 + \dots + R_n$
Find the equivalent (total) resistance in the circuit.	$R_T = 100 + 690 + 330 = 1120\ \Omega$

Use Ohm's law to calculate the current in the circuit. Whenever calculating current in a series circuit, use R_T and the potential difference of the power supply.	$I = \frac{V}{R}$ $= \frac{12}{1120}$ $= 0.011 \text{ A}$
Use Ohm's law to calculate the potential difference across each separate resistor.	$\Delta V = IR$ Therefore: $\Delta V_1 = 0.011 \text{ A} \times 100 \Omega = 1.1 \text{ V}$ $\Delta V_2 = 0.011 \text{ A} \times 690 \Omega = 7.6 \text{ V}$ $\Delta V_3 = 0.011 \text{ A} \times 330 \Omega = 3.6 \text{ V}$
Use the loop rule to check the answer.	$\Delta V_T = \Delta V_1 + \Delta V_2 + \Delta V_3$ $= 1.1 + 7.6 + 3.6$ $= 12.3 \text{ V}$ Since this is approximately the same as the potential difference provided by the cell, the answer is reasonable.

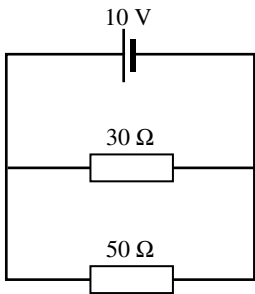
Worked example: Try yourself 5.5.3

CALCULATING AN EQUIVALENT PARALLEL RESISTANCE

A 20Ω resistor is connected in parallel with a 50Ω resistor. Calculate the equivalent parallel resistance.	
Thinking	Working
Recall the formula for equivalent effective resistance.	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$
Substitute in the given values for resistance.	$\frac{1}{R_T} = \frac{1}{20} + \frac{1}{50}$
Solve for R_T .	$\frac{1}{R_T} = \frac{1}{20} + \frac{1}{50}$ $= \frac{5}{100} + \frac{2}{100}$ $= \frac{7}{100}$ $R_T = \frac{100}{7}$ $= 14.3 \Omega$

Worked example: Try yourself 5.5.4

USING EQUIVALENT PARALLEL RESISTANCE FOR CIRCUIT ANALYSIS

Find the equivalent parallel resistance to calculate the current flowing out of the 10 V cell in the parallel circuit shown. Then find the current flowing through each resistor.	
	

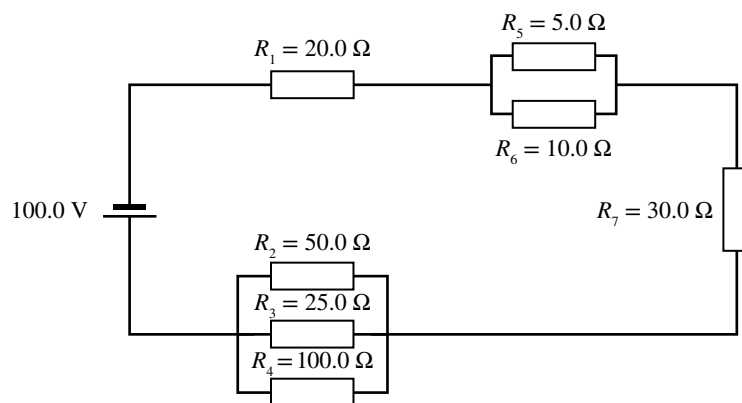
Thinking	Working
Recall the formula for equivalent parallel resistance.	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$

Substitute in the given values for resistance.	$\frac{1}{R_T} = \frac{1}{30} + \frac{1}{50}$
Solve for R_T .	$\frac{1}{R_T} = \frac{1}{30} + \frac{1}{50}$ $\frac{1}{R_T} = \frac{5}{150} + \frac{3}{150}$ $\frac{1}{R_T} = \frac{8}{150}$ $R_T = \frac{150}{8}$ $= 18.8 \Omega$
Use Ohm's law to calculate the current in the circuit. To calculate I , use the potential difference of the power supply and the total resistance.	$I_{\text{circuit}} = \frac{V}{R} = \frac{10}{18.8} = 0.53 \text{ A}$
Use Ohm's law to calculate the current through each resistor. Remember that the potential difference across each resistor is the same as the potential difference of the power supply, 10V in this case.	<p>30 Ω resistor:</p> $I_{30} = \frac{V}{R} = \frac{10}{30} = 0.33 \text{ A}$ <p>50 Ω resistor:</p> $I_{50} = \frac{V}{R} = \frac{10}{50} = 0.20 \text{ A}$
Use the junction rule to check the answers.	$I_{\text{circuit}} = I_{30} + I_{50}$ $0.53 \text{ A} = 0.33 \text{ A} + 0.20 \text{ A}$ <p>This is correct, so the answers are reasonable.</p>

Worked example: Try yourself 5.5.5

COMPLEX CIRCUIT ANALYSIS

Calculate the potential difference across and the current through each resistor in the circuit below.



Thinking	Working
Find an equivalent resistance for the two parallel resistors. The effective resistance of these should be less than the smaller resistor, that is, less than 5.0 Ω .	$\frac{1}{R_{5-6}} = \frac{1}{R_5} + \frac{1}{R_6}$ $= \frac{1}{5.0} + \frac{1}{10.0}$ $= \frac{2}{10} + \frac{1}{10}$ $R_{2-3} = \frac{10}{3} = 3.33 \Omega$
Find an equivalent resistance for the three parallel resistors. The effective resistance of these should be less than the smaller resistor, that is, less than 5.0 Ω .	$\frac{1}{R_{2,3,4}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$ $= \frac{1}{50} + \frac{1}{25} + \frac{1}{100}$ $= \frac{2}{100} + \frac{4}{100} + \frac{1}{100}$ $R_{2,3,4} = \frac{100}{7} = 14.3 \Omega$

Find an equivalent series resistance for the circuit as the circuit can now be thought of as four resistors in series: $20.0\ \Omega$, $14.3\ \Omega$, $3.33\ \Omega$ and $30.0\ \Omega$.	$R_T = 20.0\ \Omega + 14.3\ \Omega + 3.33\ \Omega + 30.0\ \Omega$ $= 67.63\ \Omega$
Use Ohm's law to calculate the current in the circuit. Use the potential difference of the power supply and the total resistance to do this calculation.	$\Delta V = IR$ $I = \frac{V}{R}$ $= \frac{100.0}{67.6}$ $= 1.48\ \text{A}$
Use Ohm's law to calculate the potential difference across each resistor (or parallel group of resistors) in series. (Note that the potential difference across R_2 is the same as that across R_3 as they are in parallel.)	$\Delta V = IR$ $\Delta V_1 = 1.48 \times 20.0 = 29.6\ \text{V}$ $\Delta V_{2,3,4} = 1.48 \times 14.3 = 21.2\ \text{V}$ $\Delta V_{5,6} = 1.48 \times 3.33 = 4.93\ \text{V}$ $\Delta V_7 = 1.48 \times 30.0 = 44.4\ \text{V}$ Check: $29.6 + 21.2 + 4.93 + 44.4 = 100.13\ \text{V}$ (with some slight rounding error = 100) This confirms that the loop rule holds for this circuit.
Use Ohm's law where necessary to calculate the current through each resistor.	$I_1 = I_7 = 1.48\ \text{A}$ $I = \frac{V}{R}$ $I_2 = \frac{21.2}{50.0} = 0.424\ \text{A}$ $I_3 = \frac{21.2}{25.0} = 0.848\ \text{A}$ $I_4 = \frac{21.2}{100} = 0.212\ \text{A}$ Check: $0.424 + 0.848 + 0.212 = 1.48\ \text{A}$ (with some slight rounding error) This confirms that the junction rule holds for this parallel section. $I_5 = \frac{4.93}{5.0} = 0.986\ \text{A}$ $I_6 = \frac{4.93}{10.0} = 0.493\ \text{A}$ Check: $0.968 + 0.493 = 1.48$ (with some slight rounding error) This confirms that the junction rule holds for this parallel section.

Worked example: Try yourself 5.5.6

COMPARING POWER IN SERIES AND PARALLEL CIRCUITS

Consider a $200\ \Omega$ and an $800\ \Omega$ resistor wired in parallel with a $12\ \text{V}$ cell. Calculate the power drawn by these resistors. Compare this to the power drawn by the same two resistors when wired in series.	
Thinking	Working
Calculate the equivalent resistance for the parallel circuit.	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$ $= \frac{1}{200} + \frac{1}{800}$ $= \frac{4}{800} + \frac{1}{800}$ $= \frac{5}{800}$ $R_T = \frac{800}{5}$ $= 160\ \Omega$

Calculate the total current drawn by the parallel circuit.	$\Delta V = IR$ $I = \frac{V}{R} = \frac{12}{160} = 0.075 \text{ A}$
Use the power equation to calculate the power drawn by the parallel circuit.	$P = \Delta VI$ $= 12 \times 0.075$ $= 0.90 \text{ W}$
Calculate the equivalent resistance for the series circuit.	$R_T = R_1 + R_2 + \dots + R_n$ $= 200 + 800$ $= 1000 \Omega$
Calculate the total current drawn by the series circuit.	$\Delta V = IR$ $I = \frac{12}{1000} = 0.012 \text{ A}$
Use the power equation to calculate the power drawn by the series circuit.	$P = \Delta VI$ $= 12 \times 0.012$ $= 0.144 \text{ W}$
Compare the power drawn by the two circuits.	$\frac{P_{\text{parallel}}}{P_{\text{series}}} = \frac{0.90}{0.144} = 6.25$ The parallel circuit draws over 6 times as much power as the series circuit.

Section 5.5 Review

KEY QUESTIONS SOLUTIONS

1 B

$$R_T = R_1 + R_2$$

$$= 20 + 20 = 40 \Omega$$

$$I = \frac{V}{R_T}$$

$$= \frac{6}{40} = 0.15 \text{ A}$$

ΔV across each resistor:

$$\Delta V = IR = 0.15 \times 20 = 3 \text{ V}$$

(Or, as the resistors are equal, the same voltage will be lost across each and will add to 6V, so 3V must be lost across each resistor.)

2 a $R_T = R_1 + R_2 + R_3$

$$= 100 + 250 + 50 = 400 \Omega$$

$$I_T = \frac{V}{R_T} = \frac{3}{400} = 0.0075 \text{ A or } 7.5 \text{ mA}$$

b $R = 100 \Omega$ and $I = 0.0075 \text{ A}$

$$\Delta V_{100} = IR$$

$$= 0.0075 \times 100 = 0.75 \text{ V}$$

3 $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_1} + \frac{1}{R_1}$ (the resistors are identical, so $R_1 = R_2$)

$$= \frac{2}{R_T}$$

$$\therefore R_T = \frac{R_1}{2}$$

$$R_1 = 2 \times R_T = 2 \times 68 = 136 \Omega$$

$$\begin{aligned}
 4 \quad \mathbf{a} \quad \frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} \\
 &= \frac{1}{20} + \frac{1}{10} \\
 &= \frac{1}{20} + \frac{2}{20} \\
 &= \frac{3}{20} \\
 R_T &= \frac{20}{3} \\
 &= 6.67 \Omega \\
 I &= \frac{\Delta V}{R} \\
 I_T &= \frac{5}{6.67} \\
 &= 0.75 \text{ A}
 \end{aligned}$$

$$\mathbf{b} \quad I_{20} = \frac{V_{20}}{R} = \frac{5}{20} = 0.25 \text{ A}$$

$$\mathbf{c} \quad I_{10} = \frac{V_{10}}{R} = \frac{5}{10} = 0.5 \text{ A}$$

$$5 \quad \mathbf{a} \quad \Delta V = IR$$

$$\Delta V_{40} = 0.3 \times 40 = 12 \text{ V (300 mA = 0.3 A)}$$

Since the components are in parallel the voltage across the 40Ω resistor (or the 60Ω resistor) is also the voltage of the battery.

$$\mathbf{b} \quad I_{60} = \frac{V_{60}}{R} = \frac{12}{60} = 0.2 \text{ A (or 200 mA)}$$

6 First determine the total resistance of the circuit:

$$\begin{aligned}
 \frac{1}{R_{3-4}} &= \frac{1}{R_3} + \frac{1}{R_4} \\
 &= \frac{1}{10} + \frac{1}{10}
 \end{aligned}$$

$$R_{3-4} = 5 \Omega$$

$$R_T = 20 + 15 + 5 = 40 \Omega$$

$$I_T = \frac{V_T}{R_T} = \frac{12}{40} = 0.3 \text{ A (or 300 mA)}$$

$$I_1 = I_2 = I_T = 0.3 \text{ A (since these are in series)}$$

$$\Delta V_1 = I_1 R_1 = 0.3 \times 20 = 6 \text{ V}$$

$$\Delta V_2 = I_2 R_2 = 0.3 \times 15 = 4.5 \text{ V}$$

$$\Delta V_3 = \Delta V_4 = I_{3-4} R_{3-4} = 0.3 \times 5 = 1.5 \text{ V}$$

$$I_3 = I_4 = \frac{V_3}{R_3} = \frac{V_4}{R_4} = \frac{1.5}{10} = 0.15 \text{ A}$$

$$7 \quad R_{\text{top row}} = 3 + 4 = 7 \Omega$$

$$R_{\text{bottom row}} = 5 + 6 = 11 \Omega$$

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{7} + \frac{1}{11} = \frac{11}{77} + \frac{7}{77} = \frac{18}{77}$$

$$R_{\text{parallel}} = 4.278 \Omega$$

8 a $R_T = R_1 + R_2 + \dots + R_n$
 $= 20 + 20 + 20 + 20$
 $= 80 \Omega$
 $\Delta V = IR$
 $\therefore I = \frac{V}{R}$
 $= \frac{10}{80}$
 $= 0.125 \text{ A}$
 $P = \Delta VI$
 $= 10 \times 0.125$
 $= 1.25 \text{ W}$

b $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \dots + \frac{1}{R_n}$
 $= \frac{1}{20} + \frac{1}{20} + \frac{1}{20} + \frac{1}{20}$
 $= \frac{4}{20}$
 $\frac{R_T}{1} = \frac{20}{4}$
 $R_T = 5 \Omega$
 $\Delta V = IR$
 $\therefore I = \frac{V}{R} = \frac{10}{5}$
 $= 2 \text{ A}$
 $P = \Delta VI$
 $= 10 \times 2$
 $= 20 \text{ W}$

- 9 C. Parallel wiring allows each appliance to be switched on and off independently (and also receive mains voltage supply).

Section 5.6 Electrical safety

Worked example: Try yourself 5.6.1

CALCULATING THE COST OF ELECTRICITY

A 2500 W iron is used for 2.5 hours. Assume the price for household electricity is 26 cents per kWh. How much would it cost (to the nearest cent) to use this iron for 2.5 hours?	
Thinking	Working
Convert the power consumption of the appliance to kW.	$\frac{2500}{1000} = 2.5$ $= 2.5 \text{ kW}$
Use the appropriate equation to multiply the power of the appliance in kW by the number of hours it operates.	$E = Pt$ $= 2.5 \times 2.5$ $= 6.25 \text{ kWh}$
Multiply the number of kWh by the cost per kWh.	Cost = 6.25×0.26 $= \$1.63$ (to two decimal places)

Section 5.6 Review

KEY QUESTIONS SOLUTIONS

- A. In the event of an electrical fault the current will rapidly increase through the zero resistance path offered by the earth connection. Once this current exceeds the fuse's rating the fuse will blow, shutting off power to the appliance.
- D. Double insulated appliances usually do not have an earth connection.
- $1 \text{ kWh} = 1000 \text{ W} \times 3600 \text{ s}$
 $= 3\,600\,000 \text{ J}$
 $10 \text{ kWh} = 10 \times 3\,600\,000$
 $= 3.6 \times 10^7 \text{ J}$
- This air conditioner would cost $0.75 \times 5 \times 0.27 =$ approximately \$1 to run for 5 hours. Therefore, the figure \$10 in the statement is incorrect.
- The neutral and earth are common.
- It is much safer to place the fuse in the active circuit because then it cuts off the supply to the circuit.
- The earth stake ensures that the neutral and earth conductors are at zero potential.
- The toaster will work normally, but the connection is very unsafe because it will remain live even when switched off.
- The outer casing of the appliance could become live.
- $I = \frac{240 \text{ V}}{1.0 \times 10^5} = 2.4 \text{ mA}$

CHAPTER 5 REVIEW

- $n_e = \frac{q}{q_e}$
 $= \frac{-3}{-1.6 \times 10^{-19}}$
 $= 1.9 \times 10^{19} \text{ electrons}$
- $q = n_e \times q_e$
 $= 4.2 \times 10^{19} \times 1.6 \times 10^{-19} \text{ C}$
 $= 6.7 \text{ C}$
- A. In a solid metal, electrons are the only charged particles that are free to move. Electrons are negatively charged.
- $q = n_e \times q_e$
 $= 2 \times 1.6 \times 10^{-19} \text{ C}$
 $= 3.2 \times 10^{-19} \text{ C}$
- $I = \frac{q}{t}$
 $= \frac{0.23}{60}$
 $= 3.8 \times 10^{-3} \text{ A}$
- Conventional current represents the flow of charge around a circuit as if the moving charges were positive, which means the direction is from the positive terminal to the negative terminal. In reality, the moving particles in a metal wire are negatively charged electrons. Electron flow describes the movement of these electrons from the negative terminal to the positive terminal.
- a** $q = It$

 $= 1.6 \times 100$
 $= 160 \text{ C}$

b $n_e = \frac{q}{q_e}$

 $= \frac{160}{1.6 \times 10^{-19}}$
 $= 10^{21} \text{ electrons}$

$$\begin{aligned}
 \mathbf{8} \quad \mathbf{a} \quad q &= n_e \times q_e \\
 &= 5 \times 10^{18} \times 1.6 \times 10^{-19} \\
 &= 0.8 \text{ C}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad t &= \frac{q}{I} \\
 &= \frac{0.8}{0.04} \\
 &= 20 \text{ seconds}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{9} \quad E &= \Delta Vq \\
 &= 3.8 \times 2 \\
 &= 7.6 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{10} \quad V &= \frac{E}{q} \\
 &= \frac{2}{0.5} \\
 &= 4 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{11} \quad P &= \frac{E}{t} \\
 &= \frac{2500}{30 \times 60} \\
 &= 1.39 \text{ W}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{12} \quad V &= \frac{E}{q} \\
 &= \frac{1.4 \times 10^{-18}}{1.6 \times 10^{-19}} \\
 &= 8.75 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{13} \quad I &= \frac{P}{V} \\
 &= \frac{2000}{230} \\
 &= 8.7 \text{ A}
 \end{aligned}$$

14 At 50V, $I = 150 \text{ mA} = 0.15 \text{ A}$

$$\begin{aligned}
 R &= \frac{V}{I} \\
 &= \frac{50}{0.15} \\
 &= 333 \Omega
 \end{aligned}$$

15 A. The equivalent resistance of resistors in series is the sum of their individual resistances.

$$\begin{aligned}
 \mathbf{16} \quad \mathbf{a} \quad R_T &= R_{\text{parallel pair}} + R_3 \\
 \therefore R_3 &= R_T - R_{\text{parallel pair}} = 8.5 - 5 = 3.5 \Omega
 \end{aligned}$$

$$\mathbf{b} \quad I_3 = I_T = \frac{V_T}{R_T} = \frac{3}{8.5} = 0.35 \text{ A}$$

$$\begin{aligned}
 \mathbf{c} \quad \Delta V_3 &= I_3 \times R_3 = 0.35 \times 3.5 = 1.2 \text{ V} \\
 \Delta V_{\text{parallel pair}} &= 3 - 1.2 = 1.8 \text{ V}
 \end{aligned}$$

$$\mathbf{d} \quad I_2 = \frac{V_2}{R_2} = \frac{1.8}{15} = 0.12 \text{ A}$$

$$\mathbf{e} \quad I_1 = I_T - I_2 = 0.35 - 0.12 = 0.23 \text{ A}$$

$$\mathbf{f} \quad R_1 = \frac{V_1}{I_1} = \frac{1.8}{0.23} = 7.83 \Omega$$

17 a Ammeter. The meter is connected in series so it must be an ammeter.

$$\begin{aligned} \text{b } \frac{1}{R_{\text{top parallel group}}} &= \frac{1}{40} + \frac{1}{40} = \frac{2}{40} \\ R_{\text{top parallel group}} &= 20 \Omega \\ \frac{1}{R_{\text{bottom parallel group}}} &= \frac{1}{20} + \frac{1}{60} = \frac{3}{60} + \frac{1}{60} = \frac{4}{60} \\ R_{\text{bottom parallel group}} &= 15 \Omega \\ \frac{1}{R_{\text{total}}} &= \frac{1}{20} + \frac{1}{15} \\ &= \frac{3}{60} + \frac{4}{60} = \frac{7}{60} \\ R_{\text{total}} &= \frac{60}{7} = 8.57 \Omega \end{aligned}$$

18 The earth wire is usually connected to the metal casing of an electrical appliance. If the insulation around the wire inside the appliance becomes degraded, the casing of the appliance could become live and dangerous to touch. In this situation, the earth wire provides an alternative low-resistance path to earth, protecting users of the appliance from electrocution.

19 The circuit will need to have either two pairs of series resistors connected in parallel or two pairs of parallel resistors connected in series.

$$20 \text{ a } R_T = R_1 + R_2 + R_3 = 20 + 20 + 20 = 60 \Omega$$

$$\Delta V = IR$$

$$\therefore I = \frac{V}{R} = \frac{12}{60} = 0.2 \text{ A}$$

$$P = \Delta VI = 12 \times 0.2 = 2.4 \text{ W}$$

$$\text{b } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{3}{20}$$

$$R_T = \frac{20}{3} = 6.67 \Omega$$

$$I = \frac{V}{R} = \frac{12}{6.67} = 1.8 \text{ A}$$

$$P = \Delta VI = 12 \times 1.8 = 21.6 \text{ W}$$

$$\text{c } \frac{P_{\text{parallel}}}{P_{\text{series}}} = \frac{21.6}{2.4} = 9$$

The parallel circuit draws 9 times more power.

21 D. A 50 mA current for over 4.5 s is likely to cause severe shock and possible death.

$$22 \quad E = Pt$$

$$= 3 \times 4$$

$$= 12 \text{ kWh}$$

$$\text{Cost} = 12 \times 0.30$$

$$= \$3.60$$

23 D. Power is a measure of how quickly energy is consumed/supplied/transformed.

$$1 \text{ watt} = 1 \text{ joule per second}$$

$$24 \text{ a } \frac{1}{R_T} = \frac{1}{100} + \frac{1}{200} + \frac{1}{600}$$

$$= \frac{6}{600} + \frac{3}{600} + \frac{1}{600}$$

$$= \frac{10}{600}$$

$$R_T = \frac{600}{10} = 60 \Omega$$

$$\text{b } \Delta V = IR$$

$$120 = I \times 60$$

$$I = 2.0 \text{ A}$$

c Each branch has 120V across it

$$I_1 = \frac{V}{R_1} = \frac{120}{100} = 1.2 \text{ A}$$

$$I_2 = \frac{V}{R_2} = \frac{120}{200} = 0.6 \text{ A}$$

$$I_3 = \frac{V}{R_3} = \frac{120}{600} = 0.2 \text{ A}$$

$$I_1 = 1.20 \text{ A}, I_2 = 0.60 \text{ A}, I_3 = 0.20 \text{ A}$$

d $P = \Delta VI$

$$= 120 \times (1.2 + 0.6 + 0.2)$$

$$= 240 \text{ W}$$

e 240 W

25 a 4V. This is calculated using Ohm's law, or alternatively, by recognising that the voltage drop at ΔV_{out} is half the voltage drop across the LDR.

$$\Delta V = IR$$

$$12 = I \times 300$$

$$I = 0.04 \text{ A}$$

$$\Delta V = IR$$

$$= 0.04 \times 100$$

$$= 4 \text{ V}$$

b above; as light increases, the resistance of the LDR decreases, hence V_{out} rises

c V_{out} approaches zero, as the LDR has increased resistance and therefore the voltage drop across the LDR approaches 12V.

26 a Combining Ohm's law, $\Delta V = IR$, and the equation for power:

$$P = \Delta VI = I^2 R = \frac{V^2}{R}$$

$$25 = I_x^2 R_x$$

$$I_x^2 = \frac{25}{100} = 25 \times 10^{-2}$$

$$I_x = 5 \times 10^{-1} = 0.5 \text{ A}$$

$$\Delta V_x = IR_x = 0.5 \times 100 = 50 \text{ V}$$

$$\Delta V_y = R_y \times \frac{1}{2} \times 0.5 = 100 \times 0.25 = 25 \text{ V}$$

$$\Delta V_{\text{total}} = \Delta V_x + \Delta V_y = 75 \text{ V}$$

b $P = \Delta VI = 75 \times 0.5 = 37.5 \text{ W}$

27 a 3

b 1

c 2

28 The finger provides less contact with the live wire and hence more resistance.

29 A fuse will melt when a high current flows in a circuit. Without the fuse the heat generated from a high current could be enough to start a fire and burn the house down. A safety switch switches off a circuit when the current in the active and neutral wires are not equal, thus preventing possible electrocution.

Unit 1 REVIEW

- 1 $Q_{\text{lost hot water}} = Q_{\text{gain cold water}}$
 $mc\Delta T_{\text{hw}} = mc\Delta T_{\text{cw}}$
 $m\epsilon(T_{\text{ihw}} - T_f) = m\epsilon(T_f - T_{\text{icw}})$
 $2T_f = T_{\text{icw}} + T_{\text{ihw}}$
 $T_f = \frac{80.0 + 10.0}{2}$
 $= 45.0^\circ$
- 2 **a** Correct. The water in the wet cloth requires energy to evaporate. It obtains this latent heat of evaporation from the bottle and its contents, thereby decreasing the temperature of the contents.
b Incorrect. The temperature of the boiling water will remain constant. All heat added during this time is causing the water to change state not temperature.
c Correct. Obviously the amount of steam or water in question would make a difference since the heat is proportional to the mass.
 If the masses are equal, the steam will burn more severely because of the additional latent heat that is released when it condenses to water on the person's skin at 100°C .
- 3 **a** A fuse protects against overload current in the total circuit. It prevents overheating of the wiring due to excess current as this poses a fire hazard. An RCD detects an imbalance between current entering and leaving a device, which suggests there is some earth leakage with that current flowing to earth. Both will shut down the circuit.
b A short circuit is a fault in the circuit that connects the active and neutral wires, effectively bypassing the load in the circuit. This means there is a greatly reduced resistance due to the absence of a load, causing a high current to flow. This condition will trigger the circuit breaker.
c Plugs with three prongs have an active, a neutral and an earth pin. The connection of an earth is required when there is any possibility that the active lead could contact the metal casing of an appliance and risk electrocution of the user as they become the contact to earth. Some smaller devices are double insulated and so the active wire cannot deliver charge to any part of the device that a user can touch. In this case, the earth is not needed and the plug can safely have only two prongs.
- 4 **a** ${}^7_3\text{Li} + {}^1_1\text{H} \rightarrow 2{}^4_2\text{He}$
b ${}^{185}_{79}\text{Au} \rightarrow {}^4_2\text{He} + {}^{181}_{77}\text{Ir}$
c ${}^{218}_{81}\text{Tl} \rightarrow {}^{218}_{82}\text{Pb} + {}^0_{-1}\text{e}$
- 5 **a** Electrostatic forces of repulsion act on the protons. They do not have enough energy to overcome this force to get close enough for the strong nuclear force to come into effect and hence will not fuse. These protons have not jumped the energy barrier.
b Electrostatic forces of repulsion act on the two protons initially, but the protons have enough energy to push past these forces and get close enough together for the strong nuclear forces to take effect. This force enables the nucleons to fuse. These protons have overcome the energy barrier.
- 6 **a** The energy of the photons results from the conversion of the mass of the electron and positron to energy.
b $E_{\text{comb}} = m_{\text{e}^-}c^2 + m_{\text{e}^+}c^2$
 $= (9.11 \times 10^{-31}) \times (3.00 \times 10^8)^2 + (9.11 \times 10^{-31}) \times (3.00 \times 10^8)^2$
 $= 1.64 \times 10^{-13} \text{ J}$
 $= \frac{(1.64 \times 10^{-13})}{(1.60 \times 10^{-19})}$
 $= 1.03 \times 10^6 \text{ eV}$
 $= 1.03 \text{ MeV}$
- 7 **a** The binding energy of a nucleus is the energy that would be needed to break the nucleus into its component nucleons. The binding energy per nucleon is this total value divided by the number of nucleons in the nucleus.
b From the graph, it can be seen that iron atoms have the highest binding energy per nucleon. Iron atoms require the most energy per nucleon to break up their nucleus, therefore they are the most stable.
c The energy per nucleon for uranium is about 7.5 MeV and the binding energy per nucleon for fragments of mass number 118 is about 8.5 MeV . That means that when the smaller fragments are formed, they are more tightly bound and the difference in energy is released in the fission reaction. This is about 1 MeV for each nucleon.

8 Power input to motor = $VI = 6.0 \times 0.25 = 1.5 \text{ W}$

Power output = $\frac{4.0}{5.0} = 0.8 \text{ W}$

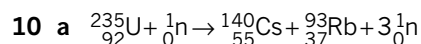
Efficiency = $\frac{0.8}{1.5} \times 100 = 53\%$

9 a $Q_{vN} = m_N L_{vN}$
 $= 1.00 \times (1.99 \times 10^5)$
 $= 1.99 \times 10^5 \text{ J}$

b $Q_{\text{gain N}} = m_N c_N \Delta T_N$
 $= 1.00 \times (1.34 \times 10^3) \times (273.0 - 77.0)$
 $= 2.63 \times 10^5 \text{ J}$

c $Q_{\text{lost s+c}} = m_{s+c} c_{s+c} \Delta T_{s+c} + m_w L_{fw}$
 $= 0.200 \times (3.80 \times 10^3) \times (8.0 - 0.0) + (0.70 \times 0.200) \times (3.34 \times 10^5)$
 $= 6.08 \times 10^3 + 4.676 \times 10^4$
 $= 5.28 \times 10^4 \text{ J}$

d $Q_{\text{lost s+c}} = Q_{\text{gain}}$
 $m_{s+c} c_{s+c} \Delta T_{s+c} + m_w L_{fw} = m_N c_N \Delta T_N + m_N L_{vN}$
 $5.28 \times 10^4 = m_N \times (1.34 \times 10^3) \times (273.0 - 77.0) + m_N (1.99 \times 10^5)$
 $5.28 \times 10^4 = (2.626 \times 10^5 m_N) + (1.99 \times 10^5 m_N)$
 $m_N = \frac{5.28 \times 10^4}{4.6164 \times 10^5}$
 $= 0.114 \text{ kg}$



b $m_{\text{reactants}} = 235.07295 + 1.00899$
 $= 236.08194 \text{ u}$

$m_{\text{products}} = 139.96265 + 92.95241 + 3 \times (1.00899)$
 $= 235.94203 \text{ u}$

$\Delta m = m_{\text{reactants}} - m_{\text{products}}$
 $= 236.08194 - 235.94203$
 $= 0.13991 \text{ u}$

$E = \text{u} \times 931$
 $= 0.13991 \times 931$
 $= 130 \text{ MeV}$
 $= 1.30 \times 10^8 \text{ eV}$

$E = (1.30 \times 10^8) \times (1.60 \times 10^{-19})$
 $= 2.08 \times 10^{-11} \text{ J}$

Alternative solution:

$m_{\text{reactants}} = (3.90221 \times 10^{-25}) + (1.67493 \times 10^{-27})$
 $= 3.918959 \times 10^{-25} \text{ kg}$

$m_{\text{products}} = (2.32338 \times 10^{-25}) + (1.54301 \times 10^{-25}) + 3 \times (1.67493 \times 10^{-27})$
 $= 3.916638 \times 10^{-25} \text{ kg}$

$\Delta m = m_{\text{reactants}} - m_{\text{products}}$
 $= (3.918959 \times 10^{-25}) - (3.916638 \times 10^{-25})$
 $= 2.32140 \times 10^{-28} \text{ kg}$

$E = \Delta mc^2$
 $= (2.32140 \times 10^{-28}) \times (3.00 \times 10^8)^2$
 $= 2.09 \times 10^{-11} \text{ J}$

$E = \frac{2.09 \times 10^{-11}}{1.60 \times 10^{-19}}$
 $= 1.31 \times 10^8 \text{ eV}$

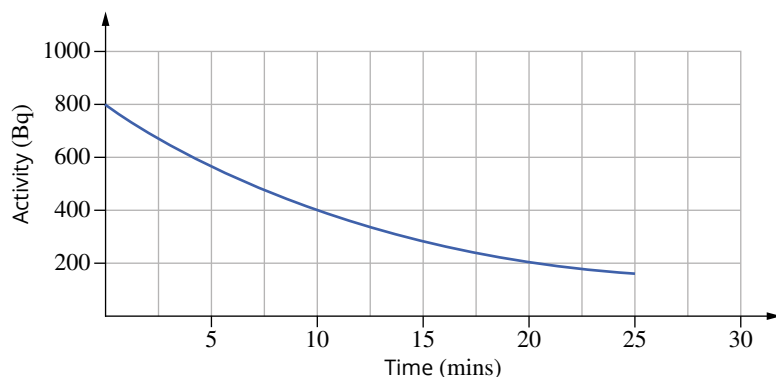
$$\text{c } N_{\text{U-235}} = \frac{1.00}{3.90221 \times 10^{-25}} = 2.56265 \times 10^{24} \text{ atoms}$$

$$\begin{aligned} E_{\text{total}} &= N_{\text{U-235}} \times E \\ &= (2.56265 \times 10^{24}) \times (2.09 \times 10^{-11}) \\ &= 5.36 \times 10^{13} \text{ J} \end{aligned}$$

$$\begin{aligned} \text{d } m_{\text{U-235}} &= \frac{E_{\text{per day}}}{E_{\text{per kg}}} \\ &= \frac{(9.76 \times 10^{13})}{(5.35 \times 10^{13})} \\ &= 1.82 \text{ kg} \end{aligned}$$

- e The conversion of energy released in the reaction to the final generation of electricity is fairly inefficient and has many losses. The energy released in the fission reactions as heat must first be used to heat up water to produce steam to drive the generators. There are many opportunities for energy losses in this system.

11 a



- b From the graph, after 13 minutes, the activity is about 320 Bq.
 c Find the time at which activity has been reduced from 800 Bq to 400 Bq:

$$t_{\frac{1}{2}} \approx 10 \text{ min}$$

- d From the graph, extrapolate to find the activity when $t = 30$ min.
 Activity ≈ 100 Bq

$$\text{12 a } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{100} + \frac{1}{200} + \frac{1}{600} = R_T = 60 \Omega$$

$$\text{b } I = \frac{\Delta V}{R} = \frac{120}{60} = 2.0 \text{ A}$$

$$\text{c } I_1 = \frac{\Delta V}{R_1} = \frac{120}{100} = 1.20 \text{ A}, I_2 = \frac{\Delta V}{R_2} = \frac{120}{200} = 0.60 \text{ A}, I_3 = \frac{\Delta V}{R_3} = \frac{120}{600} = 0.20 \text{ A}$$

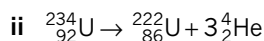
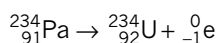
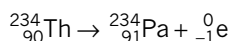
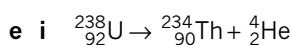
$$\text{d } P = \Delta VI = 120 \times 2.0 = 240 \text{ W}$$

$$\text{e } P = I_2 R_1 + I_2 R_2 + I_2 R_3 = 1.20^2 \times 100 + 0.60^2 \times 200 + 0.20^2 \times 600 = 240 \text{ W}$$

- 13 a This is the naturally occurring radiation that is around us every day. It can come from the Sun, outer space, materials in the Earth's crust and the food we eat.
 b The burning of coal (as well as other fossil fuels) releases radioactive materials into the atmosphere that are normally locked into the structure of the solid coal.
 c They are further outside of the protective atmospheric layers of the Earth so less radiation is absorbed before reaching them. The atmosphere becomes less dense the higher you go, so the shielding effect the atmosphere has on the incident radiation is less.

d

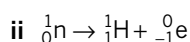
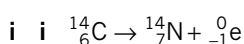
Type of radiation	Description	Name three types	Name one source for each
Ionising	This is a particle or electromagnetic radiation with enough energy per particle of photon to remove electrons from an atom or molecule and produce an ion.	alpha particles	nuclear decay
		beta particles	nuclear decay
		gamma rays	cosmos—hottest objects such as neutron stars, pulsars and black holes
		X-rays	cosmic radiation or X-ray machines
		high-frequency ultraviolet	solar radiation and electric arcs
Non-ionising	This is electromagnetic radiation with insufficient energy per photon to ionise an atom or molecule.	low-frequency radio waves	starlight two-way radio, TV and radio stations
		microwaves	mobile phones, microwave ovens
		infrared	any object emitting heat (above zero K)
		visible light	Sun, gas discharge tubes, light bulbs, LEDs
		low-frequency ultraviolet	Sun



f In any uranium mine radon gas must be present. Therefore it would be safer for workers in an open cut mine as this gas would be able to escape more readily.

g Bore water comes up from underground where it has been in contact with rock containing uranium that decays to release radon. The radon will be dissolved in the water under pressure and released when the water comes to the surface. If the water is heated the radon will be less soluble and come out of solution more readily.

h $\text{dose} = 5 \times 30 \times (3.7 \times 10^{-6})$
 $= 5.55 \times 10^{-4} \text{ Sv}$




iii Carbon dating relies on measuring the concentration of carbon-14 atoms relative to the total carbon in a fossil. The basis of this is that the total carbon in the fossil will remain constant but the concentration of carbon-14 will decrease as the fossil ages, due to its decay. Knowing the half-life and relative amount of carbon-14 present at any time enables the age of the fossil to be determined.

Chapter 6 Scalars and vectors

Section 6.1 Scalars and vectors

Worked example: Try yourself 6.1.1

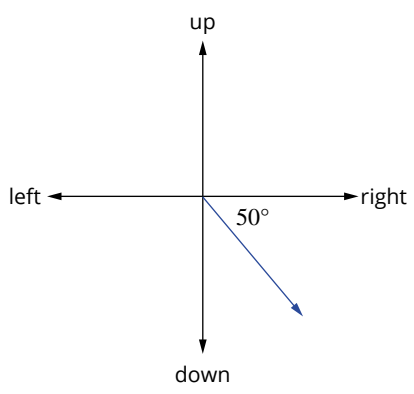
DESCRIBING VECTORS IN ONE DIMENSION

west ← → east - + 	
Describe the vector using:	
a the direction convention shown	
Thinking	Working
Identify the direction convention being used in the vector.	In this case the vector is pointing to the west according to the direction convention.
Note the magnitude, unit and direction of the vector.	In this example the vector is 50 N west.

b an appropriate sign convention.	
Thinking	Working
Convert the physical direction to the corresponding mathematical sign.	The direction west is negative.
Represent the vector with a mathematical sign, magnitude and unit.	This vector is -50 N .

Worked example: Try yourself 6.1.2

DESCRIBING TWO-DIMENSIONAL VECTORS

Describe the direction of the following vector using an appropriate method.	
	
Thinking	Working
Choose the appropriate points to reference the direction of the vector. In this case using the horizontal reference makes more sense, as the angle is given from the horizontal.	The vector can be referenced to the horizontal.
Determine the angle between the reference direction and the vector.	From the right direction to the vector there is an angle of 50° .
Determine the direction of the vector from the reference direction.	From the right direction, the vector is down.
Describe the vector using the sequence: angle, clockwise or anticlockwise from the reference direction.	This vector is 50° down from horizontal to the right.

Section 6.1 Review

KEY QUESTIONS SOLUTIONS

- Scalar measures require a magnitude (size) and units.
- Vectors require a magnitude, units and a direction.

3

Scalar	Vector
time	force
distance	acceleration
volume	position
speed	displacement
temperature	momentum
	velocity

- If the shortest arrow is 2.7 N, the middle length arrow is twice the length of the shortest (5.4 N) and the longest is three times the shortest (8.1 N). The 9.0 N magnitude is not required.
- If the shortest arrow is -5.4 N, the middle length arrow is twice the length of the shortest (10.8 N) and the longest is three times the shortest (16.2 N). The -2.7 N magnitude is not required.
- down
 - south
 - forwards
 - up
 - east
 - positive
- Terms like north and left cannot be used in a calculation. + and - can be used to do calculations with vectors.
- The vector diagram shows -35 N.
- 225° T
 - S 45° W
 - 120° T
 - S 60° E
- 40° up from horizontal to the left

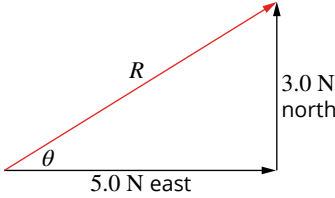
Section 6.2 Adding vectors in one and two dimensions

Worked example: Try yourself 6.2.1

ADDING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 6.2.2 to determine the resultant force on a box that has the following forces acting on it: 16 N up, 22 N down, 4 N up and 17 N down.	
Thinking	Working
Apply the sign and direction conventions to change the directions to signs.	16 N up = +16 N 22 N down = -22 N 4 N up = +4 N 17 N down = -17 N
Add the magnitudes and their signs together.	Resultant force = (+16) + (-22) + (+4) + (-17) = -19 N
Refer to the sign and direction convention to determine the direction of the resultant force vector.	Negative is down. ∴ Resultant force = 19 N down

Worked example: Try yourself 6.2.2
ADDING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the resultant force when forces of 5.0 N east and 3.0 N north act on a tree. Refer to Figure 6.2.2 on page 181 for sign and direction conventions if required.	
Thinking	Working
Construct a vector diagram showing the vectors drawn head to tail. Draw the resultant vector from the tail of the first vector to the head of the last vector.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the resultant vector.	$R^2 = 5.0^2 + 3.0^2$ $= 25 + 9$ $R = \sqrt{34}$ $= 5.8 \text{ N}$
Using trigonometry, calculate the angle from the east vector to the resultant vector.	$\tan \theta = \frac{3.0}{5.0}$ $\theta = \tan^{-1} 0.6$ $= 31.0^\circ$
Determine the direction of the vector relative to north or south.	$90^\circ - 31^\circ = 59^\circ$ The direction is N 59° E
State the magnitude and direction of the resultant vector.	$R = 5.8 \text{ N, N } 59^\circ \text{ E}$

Section 6.2 Review

KEY QUESTIONS SOLUTIONS

- Total weekly distance = $5 \times 2 \times 3.0 = 30 \text{ km}$
 - Since he returns home each day, his displacement is zero each day and each week.
- Using sign conventions, resultant = $+3 - 2 - 3 = -2$. The resultant vector is 2 m down.
- Using sign conventions, resultant = $+23 + (-16) + 7 + (-3) = +11$. The resultant vector is 11 m forwards.
- D. Adding vector B to vector A is equivalent to saying $A + B$. Therefore, draw vector A first, then draw vector B with its tail at the head of A. The resultant is drawn from the tail of the first vector (A) to the head of the last vector (B).
- $$R^2 = 40.0^2 + 20.0^2$$

$$= 1600 + 400$$

$$R = \sqrt{2000}$$

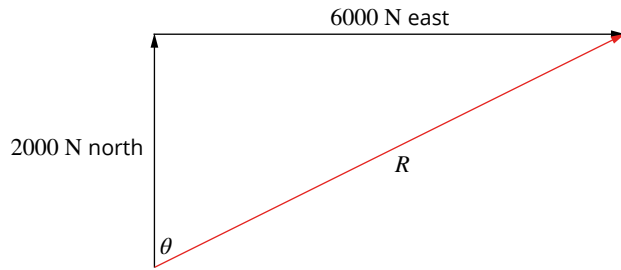
$$= 44.7 \text{ m}$$

$$\tan \theta = \frac{40.0}{20.0}$$

$$\theta = \tan^{-1} 2.00$$

$$= 63.4^\circ$$

$$R = 44.7 \text{ m, S } 63.4^\circ \text{ W}$$

6


$$R^2 = 2000^2 + 6000^2$$

$$= 4\,000\,000 + 36\,000\,000$$

$$R = \sqrt{40\,000\,000}$$

$$= 6325 \text{ N}$$

$$\tan \theta = \frac{6000}{2000}$$

$$\theta = \tan^{-1} 3.00$$

$$= 71.6^\circ$$

$$R = 6325 \text{ N, N } 71.6^\circ \text{ E}$$

7

$$R^2 = 40.0^2 + 30.0^2$$

$$= 1600 + 900$$

$$R = \sqrt{2500}$$

$$= 50.0 \text{ m}$$

8

First add 3000 N north and 5000 N south

Resultant force is 2000 N south

Then add 2000 N south to 5000 N east

$$F^2 = 2000^2 + 5000^2$$

$$= 29\,000\,000$$

$$F = 5385 \text{ N}$$

$$\theta = \tan^{-1} \frac{5000}{2000} = 68.2^\circ$$

Resultant force = 5385 N S 68.2° E

9

Total forwards force = 3350 + 2235 + 634 = 6219 N

Apply a sign convention: forwards = + 6219 N; backward = -6220 N

Add vectors = +6219 - 6220 N = -1 N

Resultant force = 1 N backwards

Section 6.3 Subtracting vectors in one and two dimensions

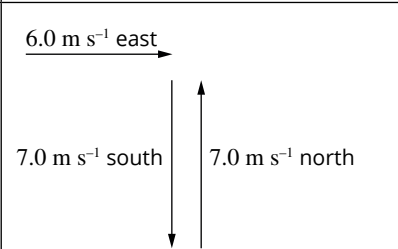
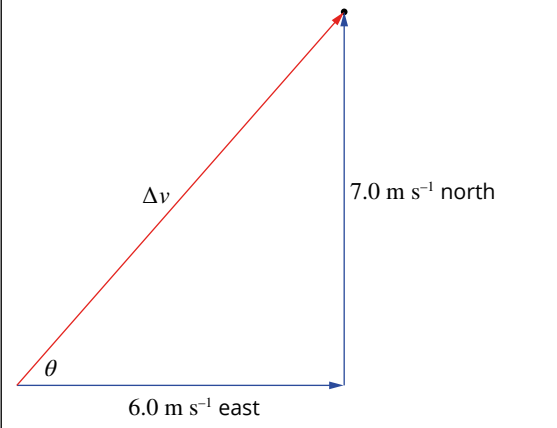
Worked example: Try yourself 6.3.1

SUBTRACTING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 6.3.5 on page 188 of the Student Book to determine the change in velocity of a rocket as it changes from 212 m s^{-1} up to 2200 m s^{-1} up.	
Thinking	Working
Apply the sign and direction conventions to change the directions to signs.	$v_1 = 212 \text{ m s}^{-1}$ up = $+212 \text{ m s}^{-1}$ $v_2 = 2200 \text{ m s}^{-1}$ up = $+2200 \text{ m s}^{-1}$
Reverse the direction of the initial velocity, v_1 , by reversing the sign.	$-v_1 = 212 \text{ m s}^{-1}$ down = -212 m s^{-1}
Use the formula for change in velocity to calculate the magnitude and the sign of Δv .	$\Delta v = v_2 + (-v_1)$ = $+2200 + (-212)$ = $+1988 \text{ m s}^{-1}$
Refer to the sign and direction convention to determine the direction of the change in velocity.	Positive is up $\therefore \Delta v = 1988 \text{ m s}^{-1}$ up

Worked example: Try yourself 6.3.2

SUBTRACTING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the change in velocity of a ball as it bounces off a wall. The ball approaches at 7.0 m s^{-1} south and rebounds at 6.0 m s^{-1} east.	
Thinking	Working
Draw the final velocity vector, v_2 , and the initial velocity vector, v_1 , separately. Then draw the initial velocity in the opposite direction.	
Construct a vector diagram, drawing v_2 first, and then from its head draw the opposite of v_1 . The change in velocity vector is drawn from the tail of the final velocity to the head of the opposite of the initial velocity.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the change in velocity.	$R^2 = 7.0^2 + 6.0^2$ $= 49 + 36$ $R = \sqrt{85}$ $= 9.2 \text{ m s}^{-1}$

Calculate the angle from the north vector to the change in velocity vector.	$\tan \theta = \frac{7.0}{6.0}$ $\theta = \tan^{-1} 1.17$ $= 49.40^\circ$ Direction from north vector is $90 - 49.40 = 40.60^\circ$
State the magnitude and direction of the change in velocity.	$\Delta v = 9.2 \text{ ms}^{-1} \text{ N } 41^\circ \text{ E}$

Section 6.3 Review

KEY QUESTIONS SOLUTIONS

1 Change in velocity = final velocity – initial velocity
 $= 5 + (+3)$
 $= 8 \text{ ms}^{-1} \text{ east}$

2 Change in velocity = final velocity – initial velocity
 $= 2 + (-4)$
 $= 2 \text{ ms}^{-1} \text{ left}$

3 Change in velocity = final velocity – initial velocity
 $= -3 + (-4)$
 $= 7 \text{ ms}^{-1} \text{ downwards}$

4 Change in velocity = final velocity – initial velocity
 $= -32.5 + (-35.0)$
 $= 67.5 \text{ ms}^{-1} \text{ south}$

5 Change in velocity = final velocity – initial velocity
 $= 8.2 + (-22.2)$
 $= 14.0 \text{ ms}^{-1} \text{ backwards}$

6 $\Delta v^2 = (v_2)^2 + (-v_1)^2$
 $= (406)^2 + (345)^2$
 $\Delta v = \sqrt{1648.36 + 1190.25}$
 $= \sqrt{2838.61}$
 $= 533 \text{ ms}^{-1}$

$$\tan \theta = \frac{345}{406}$$

$$\theta = \tan^{-1} \frac{345}{406}$$

$$= 40.4^\circ$$

Angle measured from the north = $90^\circ - 40.4^\circ = 49.6^\circ$

$\Delta v = 533 \text{ ms}^{-1} \text{ N } 49.6^\circ \text{ W}$

7 $\Delta v^2 = (v_2)^2 + (-v_1)^2$
 $= (42.0)^2 + (42.0)^2$
 $\Delta v = \sqrt{1764 + 1764}$
 $= \sqrt{3528}$
 $= 59.4 \text{ ms}^{-1}$

$$\tan \theta = \frac{42.0}{42.0}$$

$$\theta = \tan^{-1} (1.000)$$

$$= 45.0^\circ$$

$\Delta v = 59.4 \text{ ms}^{-1} \text{ N } 45.0^\circ \text{ W}$

$$\begin{aligned}
 8 \quad \Delta v^2 &= (v_2)^2 + (-v_1)^2 \\
 &= (5.25)^2 + (7.05)^2 \\
 \Delta v &= \sqrt{(27.56 + 49.70)} \\
 &= \sqrt{77.27} \\
 &= 8.79 \text{ ms}^{-1} \\
 \tan \theta &= \frac{7.05}{5.25} \\
 \theta &= \tan^{-1} \frac{7.05}{5.25} \\
 &= 53.3^\circ
 \end{aligned}$$

Angle measured from the north = $90^\circ - 53.3^\circ = 36.7^\circ$

$$\Delta v = 8.79 \text{ ms}^{-1} \text{ N } 36.7^\circ \text{ W}$$

- 9 a $40 - 25 = 15 \text{ km h}^{-1}$
 b $25 - (-40) = 25 + 40 = 65 \text{ km h}^{-1}$ i.e. 65 km h^{-1} south

$$\begin{aligned}
 10 \quad v^2 &= (v_2)^2 + (-v_1)^2 \\
 &= (30)^2 + (30)^2 \\
 v &= \sqrt{900 + 900} \\
 &= \sqrt{1800} \\
 &= 42.4 \text{ km h}^{-1}
 \end{aligned}$$

$$\tan \theta = \frac{30}{30}$$

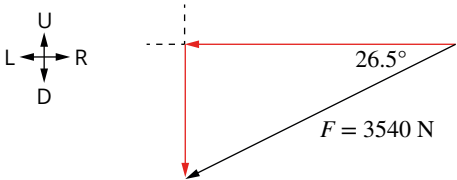
$$\begin{aligned}
 \theta &= \tan^{-1}(1) \\
 &= 45^\circ
 \end{aligned}$$

$$v = 42.4 \text{ km h}^{-1} \text{ N}45^\circ \text{ W}$$

Section 6.4 Vector components

Worked example: Try yourself 6.4.1

CALCULATING THE PERPENDICULAR COMPONENTS OF A FORCE

Use the direction conventions to determine the perpendicular components of a 3540 N force acting on a trolley at a direction of 26.5° down from horizontal to the left.	
Thinking	Working
Draw F_L from the tail of the 3540 N force along the horizontal, then draw F_D from the horizontal line to the head of the 3540 N force.	
Calculate the left component of the force F_L using $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\text{adj} = \text{hyp} \cos \theta$ $F_L = 3540 \times \cos 26.5^\circ$ $= 3168 \text{ N left}$
Calculate the downwards component of the force F_D using $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\text{opp} = \text{hyp} \sin \theta$ $F_D = 3540 \times \sin 26.5^\circ$ $= 1580 \text{ N downwards}$

Section 6.4 Review

KEY QUESTIONS SOLUTIONS

1 a $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\text{opp} = \text{hyp} \sin \theta$
 $F_D = 462 \times \sin 35.0^\circ$
 $= 265 \text{ N downwards}$

b $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\text{adj} = \text{hyp} \cos \theta$
 $F_R = 462 \times \cos 35.0^\circ$
 $= 378 \text{ N right}$

2 $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\text{adj} = \text{hyp} \cos \theta$
 $F_S = 25.9 \times \cos 40.0^\circ$
 $= 19.8 \text{ N south}$

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\text{opp} = \text{hyp} \sin \theta$
 $F_E = 25.9 \times \sin 40.0^\circ$
 $= 16.6 \text{ N east}$

Therefore, 19.8 N south and 16.6 N east

3 $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\text{adj} = \text{hyp} \cos \theta$
 $v_N = 18.3 \times \cos 75.6^\circ$
 $= 4.55 \text{ m s}^{-1} \text{ north}$

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\text{opp} = \text{hyp} \sin \theta$
 $v_W = 18.3 \times \sin 75.6^\circ$
 $= 17.7 \text{ m s}^{-1} \text{ west}$

Therefore, 4.55 m s⁻¹ north and 17.7 m s⁻¹ west

4 $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\text{adj} = \text{hyp} \cos \theta$
 $s_S = 47.0 \times \cos 66.3^\circ$
 $= 18.9 \text{ m south}$

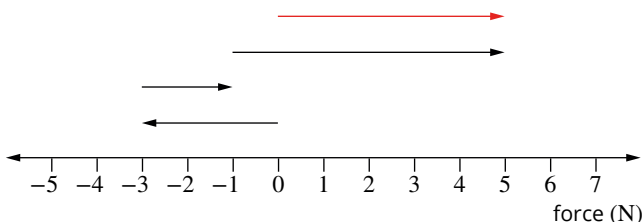
$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\text{opp} = \text{hyp} \sin \theta$
 $s_E = 47.0 \times \sin 66.3^\circ$
 $= 43.0 \text{ m east}$

Therefore, the student is 18.9 m south and 43 m east of his starting point.

- 5 $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\text{adj} = \text{hyp} \cos \theta$
 $F_N = 235\,000 \times \cos 62.5^\circ$
 $= 109\,000 \text{ N north}$
 $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\text{opp} = \text{hyp} \sin \theta$
 $F_W = 235\,000 \times \sin 62.5^\circ$
 $= 208\,000 \text{ N west}$
- 6 **a** $F_S = 100 \cos 60^\circ = 50 \text{ N south}$
 $F_E = 100 \sin 60^\circ = 87 \text{ N east}$
b $F_N = 60 \text{ N north}$
c $F_S = 300 \cos 20^\circ = 282 \text{ N south}$
 $F_E = 300 \sin 20^\circ = 103 \text{ N east}$
d $F_V = 3.0 \times 10^5 \sin 30^\circ = 1.5 \times 10^5 \text{ N up}$
 $F_h = 3.0 \times 10^5 \cos 30^\circ = 2.6 \times 10^5 \text{ N horizontal}$
- 7 horizontal component $F_h = 300 \cos 60^\circ = 150 \text{ N}$
 vertical component $F_v = 300 \sin 60^\circ = 260 \text{ N}$
- 8 vertical = $30.0 \times \sin 50.0^\circ = 23.0 \text{ ms}^{-1}$
 horizontal = $30.0 \times \cos 50.0^\circ = 19.3 \text{ ms}^{-1}$
- 9 Distance south = $340 \times \sin 45^\circ = 240 \text{ m}$
- 10 Horizontal component of force = $400 \times \cos 70.0^\circ = 137 \text{ N}$

CHAPTER 6 REVIEW

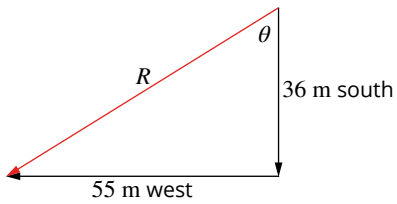
- 1 B and D are both scalars. These do not require a magnitude and direction to be fully described.
- 2 A and D are vectors. These require a magnitude and direction to be fully described.
- 3 The vector must be drawn as an arrow with its tail at the point of contact between the hand and the ball. The arrow points in the direction of the 'push' of the hand.
- 4 Vector A is drawn twice the length of vector B, so it has twice the magnitude of B.
- 5 Signs are useful in mathematical calculations, as the words north and south cannot be used in an equation.
- 6 34.0 ms^{-1} north and 12.5 ms^{-1} east. This is because the change in velocity is the final velocity plus the opposite of the initial velocity. The opposite of 34.0 ms^{-1} south is 34.0 ms^{-1} north.
- 7 + 80 N or just 80 N
- 8 70° down from horizontal to the left or 20° up from vertical to the left
- 9



The resultant vector is 5 N right.

- 10 The vectors are $(+45.0) + (-70.5) + (+34.5) + (-30.0)$. This equals -21.0 . Backwards is negative, therefore the answer is 21.0 m backwards.

11



$$R^2 = 36^2 + 55^2$$

$$= 1296 + 3025$$

$$R = \sqrt{4321}$$

$$= 65.7 \text{ m}$$

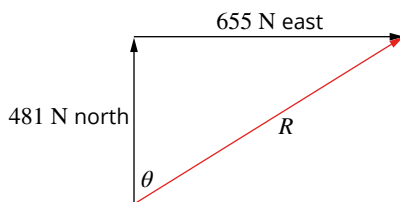
$$\tan \theta = 55 \div 36$$

$$\theta = \tan^{-1} 1.5278$$

$$= 56.8^\circ$$

Therefore, the addition of 36m south and 55m west gives a resultant vector to three significant figures of 65.7m S 56.8° W.

12



$$R^2 = 481^2 + 655^2$$

$$= 231\,361 + 429\,025$$

$$R = \sqrt{660\,386}$$

$$= 813 \text{ N}$$

$$\tan \theta = 655 \div 481$$

$$\theta = \tan^{-1} 1.3617$$

$$= 53.7^\circ$$

Therefore, the resultant vector is $R = 813 \text{ N, N } 53.7^\circ \text{ E}$.

13 Taking as positive:

$$\Delta v = v - u$$

$$= -3 + (-3)$$

$$= -6$$

$$= 6 \text{ m s}^{-1} \text{ left}$$

 14 $\Delta v^2 = (v_2)^2 + (-v_2)^2$

$$= 18.7^2 + 13.0^2$$

$$\Delta v = \sqrt{349.69 + 169}$$

$$= \sqrt{518.69}$$

$$= 22.8 \text{ m s}^{-1}$$

$$\tan \theta = \frac{18.7}{13.0}$$

$$\theta = \tan^{-1} 1.4385$$

$$= 55.2^\circ$$

$$\Delta v = 22.8 \text{ m s}^{-1} \text{ N } 55.2^\circ \text{ W}$$

- 15** $\Delta v^2 = (v_2)^2 + (-v_2)^2$
 $= 55.5^2 + 38.8^2$
 $\Delta v = \sqrt{3080.25 + 1505.4416}$
 $= \sqrt{4585.69}$
 $= 67.7 \text{ ms}^{-1}$
 $\tan \theta = \frac{38.8}{55.5}$
 $\theta = \tan^{-1} 0.6991$
 $= 35.0^\circ$
 $\Delta v = 67.7 \text{ ms}^{-1} \text{ N } 35.0^\circ \text{ W}$
- 16** $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\text{opp} = \text{hyp} \times \sin \theta$
 $F_E = 45.5 \times \sin 60.0^\circ$
 $= 39.4 \text{ N east}$
 $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\text{adj} = \text{hyp} \times \cos \theta$
 $F_S = 45.5 \times \cos 60.0^\circ$
 $= 22.8 \text{ N south}$
- 17** $u = 400 \text{ ms}^{-1}$
 $\theta = 50.0^\circ$
 $u_v = u \sin \theta$
 $= 400 \sin 50.0^\circ$
 $= 306 \text{ ms}^{-1}$
- 18** Findlay horizontal = $200 \cos 60.0^\circ = 100 \text{ N}$
 Dougie horizontal = $400 \cos 50.0^\circ = 257 \text{ N}$
 Total horizontal force = $357 \text{ N to the right}$
- 19** Note: 5.0 m distance is not needed
 Vertical component of velocity = $v \cos 20.0^\circ = 3.00$
 $v = 3.19 \text{ ms}^{-1}$
- 20** $v = 10.0 \text{ ms}^{-1}$
 $\theta = 45.0^\circ$
 $v_v = v \sin \theta = 10.0 \sin 45.0^\circ = 7.07 \text{ ms}^{-1} \text{ down}$
 $v_h = v \cos \theta = 10.0 \cos 45.0^\circ = 7.07 \text{ ms}^{-1} \text{ to the right}$

Chapter 7 Linear motion

Section 7.1 Displacement, speed and velocity

Worked example: Try yourself 7.1.1

AVERAGE VELOCITY AND CONVERTING UNITS

Sally is an athlete performing a training routine by running back and forth along a straight stretch of running track. She jogs 100 m west in a time of 20 s, then turns and walks 160 m east in a further 45 s before stopping.

a What is Sally's average velocity in m s^{-1} ?	
Thinking	Working
Calculate the displacement. Remember that total displacement is the sum of individual displacements. Sally's total journey consists of two displacements: 100 m west and 160 m east. Take east to be the positive direction.	$s = \text{sum of displacements}$ $= 100 \text{ m west} + 160 \text{ m east}$ $= -100 + 160$ $= +60 \text{ m or } 60 \text{ m east}$
Work out the total time taken for the journey.	Time taken = $20 + 45 = 65 \text{ s}$
Substitute the values into the velocity equation.	Displacement, s , is 60 m east. Time taken, t , is 65 s. Average velocity $v_{\text{av}} = \frac{s}{t}$ $= \frac{60}{65}$ $= 0.92 \text{ m s}^{-1}$
Velocity is a vector, so a direction must be given.	0.92 m s^{-1} east
b What is the magnitude of Sally's average velocity in km h^{-1} ?	
Thinking	Working
Convert from m s^{-1} to km h^{-1} by multiplying by 3.6.	$v_{\text{av}} = 0.92 \text{ m s}^{-1}$ $= 0.92 \times 3.6$ $= 3.3 \text{ km h}^{-1}$ east
As the magnitude of the velocity is needed, the direction is not required in this answer.	Magnitude of $v_{\text{av}} = 3.3 \text{ km h}^{-1}$
c What is Sally's average speed in m s^{-1} ?	
Thinking	Working
Calculate the distance. Remember that distance is the length of the path covered in the entire journey. The direction does not matter. Sally travels 100 m in one direction and then 160 m in the other direction.	$s = 100 + 160$ $= 260 \text{ m}$
Work out the total time taken for the journey.	$20 + 45 = 65 \text{ s}$
Substitute the values into the speed equation.	Distance, s , is 260 m. Time taken, t , is 65 s. Average speed $v_{\text{av}} = \frac{s}{t}$ $= \frac{260}{65}$ $= 4.0 \text{ m s}^{-1}$

d What is Sally's average speed in km h^{-1} ?

Thinking

Convert from m s^{-1} to km h^{-1} by multiplying by 3.6.

Working

$$\begin{aligned} \text{Average speed } v_{\text{av}} &= 4.0 \text{ m s}^{-1} \\ &= 4.0 \times 3.6 \\ &= 14.4 \text{ km h}^{-1} \end{aligned}$$

Section 7.1 Review

KEY QUESTIONS SOLUTIONS

1 a average speed $v_{\text{av}} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{s}{t} = \frac{400}{2 \times 60} = \frac{400}{120}$
 $= 3.33 \text{ m s}^{-1}$

b average velocity $v_{\text{av}} = \frac{\text{displacement}}{\text{time taken}} = \frac{s}{t} = \frac{0}{120}$
 $= 0.00 \text{ m s}^{-1}$

Her displacement is zero because the start and finish points are the same.

2 B and C. The distance travelled is $25 \times 10 = 250 \text{ m}$, but the displacement is zero because the swimmer starts and ends at the same place.

3 a Displacement = final position – initial position
 $= 40 - 0$
 $= +40 \text{ cm}$

Distance travelled = 40 cm

b Displacement = final position – initial position
 $= 40 - 50$
 $= -10 \text{ cm}$

Distance travelled = 10 cm

c Displacement = final position – initial position
 $= 70 - 50$
 $= 20 \text{ cm}$

Distance travelled = 20 cm

d Displacement = final position – initial position
 $= 70 - 50$
 $= 20 \text{ cm}$

Distance covered = $50 + 30$
 $= 80 \text{ cm}$

4 a $s = 50 + 30 = 80 \text{ km}$

b $s = 50 \text{ km north} + 30 \text{ km south}$
 $= 50 + (-30)$
 $= 50 - 30$
 $= +20 \text{ km or } 20 \text{ km north}$

5 a The basement is 10 m downwards from the starting position on the ground floor. This can be calculated using the following equation:

$$\begin{aligned} s &= \text{final position} - \text{initial position} \\ &= -10 - 0 \\ &= -10 \text{ m or } 10 \text{ m downwards} \end{aligned}$$

b The total displacement from the basement to the top floor is 60 m upwards. This can be calculated using the following equation:

$$\begin{aligned} s &= \text{final position} - \text{initial position} \\ &= +50 - (-10) \\ &= 50 + 10 \\ &= +60 \text{ m or } 60 \text{ m upwards} \end{aligned}$$

c The total distance travelled is 70 m.

$$10 + 10 + 50 = 70 \text{ m}$$

d The top floor is 50 m upward from the starting position on the ground floor. This can be calculated using the following equation:

$$s = \text{final position} - \text{initial position}$$

$$= 50 - 0$$

$$= 50 \text{ m or } 50 \text{ m upwards}$$

6 a average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$

$$= \frac{400}{12}$$

$$= 33 \text{ m s}^{-1}$$

b The car travelled 25 m. This can be calculated using the following method:

$$\text{average speed } v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$s = v_{av} \times t$$

$$= 33 \times 0.75$$

$$= 25 \text{ m}$$

7 a $90 \text{ min} = \frac{90}{60}$

$$= 1.5 \text{ h}$$

$$\text{average speed } v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{25}{1.5}$$

$$= 17 \text{ km h}^{-1}$$

b To convert from km h^{-1} to m s^{-1} , you need to divide by 3.6. So:

$$\text{average speed } v_{av} = \frac{17}{3.6}$$

$$= 4.7 \text{ m s}^{-1}$$

8 a average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$

$$= \frac{s}{t}$$

$$= \frac{9}{10}$$

$$= 0.9 \text{ m s}^{-1}$$

b Displacement is 1 m east of the starting position.

$$\text{average velocity } v_{av} = \frac{\text{displacement}}{\text{time taken}}$$

$$= \frac{s}{t}$$

$$= \frac{1}{10}$$

$$= 0.1 \text{ m s}^{-1} \text{ east}$$

9 a average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$

$$= \frac{2.5}{0.25}$$

$$= 10 \text{ km h}^{-1}$$

b average velocity $v_{av} = \frac{10}{3.6} = 2.8 \text{ m s}^{-1}$ south

10 a Distance travelled = 10 km north + 3 km south + x km north to finish 15 km north of the start.

$$x = 8 \text{ km north.}$$

$$\text{Total distance covered} = 10 + 3 + 8$$

$$= 21 \text{ km}$$

b She finishes 15 km north of her starting point. This is her displacement.

c average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$

$$= \frac{21}{1.5}$$

$$= 14 \text{ km h}^{-1}$$

d average velocity $v_{av} = \frac{\text{displacement}}{\text{time taken}}$

$$= \frac{15}{1.5}$$

$$v_{av} = 10 \text{ km h}^{-1}$$

Section 7.2 Acceleration

Worked example: Try yourself 7.2.1

CHANGE IN SPEED AND VELOCITY 1

A golf ball is dropped onto a concrete floor and strikes the floor at 9.0 ms^{-1} . It then rebounds at 7.0 ms^{-1} .

a What is the change in speed of the ball?		
Thinking	Working	
Find the values for the initial speed and the final speed of the ball.	$u = 9.0 \text{ ms}^{-1}$ $v = 7.0 \text{ ms}^{-1}$	
Substitute the values into the change in speed equation: $\Delta v = v - u$	$\Delta v = v - u$ $= 7.0 - 9.0$ $= -2.0 \text{ ms}^{-1}$	Note that speed is a scalar so the negative value indicates a decrease in magnitude, as opposed to a negative direction.

b What is the change in velocity of the ball?		
Thinking	Working	
Apply the sign convention to replace the directions.	$u = 9.0 \text{ ms}^{-1}$ down $= -9.0 \text{ ms}^{-1}$ $v = 7.0 \text{ ms}^{-1}$ up $= +7.0 \text{ ms}^{-1}$	
Reverse the direction of u to get $-u$.	$u = -9.0 \text{ ms}^{-1}$ $-u = 9.0 \text{ ms}^{-1}$	
Substitute the values into the change in velocity equation: $\Delta v = v + (-u)$	$\Delta v = v + (-u)$ $= 7.0 + (+9.0)$ $= 16.0 \text{ ms}^{-1}$	
Apply the sign convention to describe the direction.	$\Delta v = 16 \text{ ms}^{-1}$ up	

Worked example: Try yourself 7.2.2
CHANGE IN SPEED AND VELOCITY 2

A golf ball is dropped onto a concrete floor and strikes the floor at 9.0 m s^{-1} . It then rebounds at 7.0 m s^{-1} . The contact time with the floor is 35 ms.

What is the average acceleration of the ball during its contact with the floor?

Thinking	Working
Note the values you will need in order to find the average acceleration (initial velocity, final velocity and time). Convert ms into s by dividing by 1000. (Note that Δv was calculated for this situation in the previous Worked example.)	$u = -9.0 \text{ m s}^{-1}$ $-u = 9.0 \text{ m s}^{-1}$ $v = 7.0 \text{ m s}^{-1}$ $\Delta v = v - u = 16 \text{ m s}^{-1}$ up $t = 35 \text{ ms}$ $= 0.035 \text{ s}$
Substitute the values into the average acceleration equation.	$a = \frac{\text{change in velocity}}{\text{time taken}}$ $= \frac{\Delta v}{t} = \frac{v - u}{t}$ $= \frac{16}{0.035}$ $= 457 \text{ m s}^{-2}, \text{ which is } 460 \text{ m s}^{-2} \text{ to two significant figures}$
Acceleration is a vector, so you must include a direction in your answer.	$a = 4.6 \times 10^2 \text{ m s}^{-2}$ up

Section 7.2 Review

KEY QUESTIONS SOLUTIONS

$$\begin{aligned}
 1 \quad \Delta v &= v - u \\
 &= 3 - 10 \\
 &= -7
 \end{aligned}$$

So the change in speed is -7 km h^{-1} .

Note that speed is a scalar so the negative value indicates a decrease in magnitude, rather than a negative direction.

$$\begin{aligned}
 2 \quad \Delta v &= v - u \\
 &= 0 + (+5) \\
 &= +5 \text{ m s}^{-1} \text{ or } 5 \text{ m s}^{-1} \text{ up}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad &\text{Down is negative, so the initial velocity is } -5.0 \text{ m s}^{-1}. \\
 \Delta v &= v - u = (3.0) - (-5.0) \\
 &= +8 \text{ m s}^{-1} \\
 &= 8 \text{ m s}^{-1} \text{ up}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad a &= \frac{\text{change in velocity}}{\text{time taken}} \\
 &= \frac{v - u}{t} \\
 &= \frac{0 - 7.5}{1.5} \\
 &= -5.0 \text{ m s}^{-2} \\
 &= 5.0 \text{ m s}^{-2} \text{ south}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad a &= \frac{\text{change in velocity}}{\text{time taken}} = \frac{v - u}{t} \\
 &= \frac{150 - 0}{3.5} \\
 &= 43 \text{ km h}^{-1} \text{ s}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad a \quad \Delta v &= v - u \\
 &= 15 - 25 \\
 &= -10 \text{ m s}^{-1}
 \end{aligned}$$

Note that speed is a scalar so the negative value indicates a decrease in magnitude, as opposed to a negative direction.

$$\begin{aligned}
 b \quad \Delta v &= v - u \\
 &= (-15) - (25) \\
 &= -40 \text{ m s}^{-1} \\
 &= 40 \text{ m s}^{-1} \text{ west}
 \end{aligned}$$

$$\begin{aligned}
 c \quad a &= \frac{\text{change in velocity}}{\text{time taken}} \\
 &= \frac{v - u}{t} \\
 &= \frac{40}{0.050} \\
 &= 800 \text{ m s}^{-2}
 \end{aligned}$$

Magnitude only so the direction is not required.

$$\begin{aligned}
 7 \quad a \quad \Delta v &= v - u \\
 &= 8.0 - 0 \\
 &= 8.0 \text{ m s}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 b \quad \Delta v &= v - u \\
 &= -8.0 - 0 \\
 &= -8.0 \text{ m s}^{-1} \\
 &= 8.0 \text{ m s}^{-1} \text{ south}
 \end{aligned}$$

$$\begin{aligned}
 c \quad a &= \frac{\text{change in velocity}}{\text{time taken}} = \frac{v - u}{t} \\
 &= \frac{8.0}{1.2} \\
 &= 6.7 \text{ m s}^{-2}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad a &= \frac{v - u}{t} \\
 t &= \frac{v - u}{a} = \frac{30.0 - 10.0}{3.00} \\
 &= 6.67 \text{ s}
 \end{aligned}$$

$$\begin{aligned}
 9 \quad a &= \frac{v - u}{t} \\
 t &= \frac{v - u}{a} = \frac{00.0 - 20.0}{-2.50} \\
 &= 8.00 \text{ s}
 \end{aligned}$$

$$\begin{aligned}
 10 \quad a &= -3.00 \text{ m s}^{-2} \\
 t &= 4.00 \text{ s} \\
 v &= 0.00 \text{ m s}^{-2} \text{ (cyclist is stopped)} \\
 u &= ? \\
 v &= u + at \\
 u &= v - at = 0.00 - (-3.00) \times (4.00) \\
 &= 12.0 \text{ m s}^{-1}
 \end{aligned}$$

Section 7.3 Graphing position, velocity and acceleration over time

Worked example: Try yourself 7.3.1

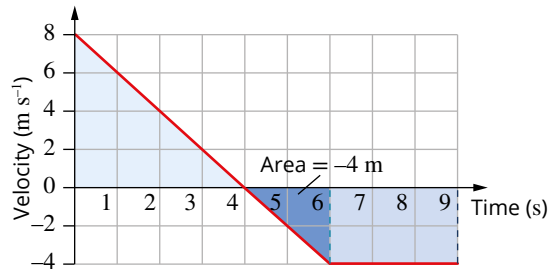
ANALYSING A POSITION–TIME GRAPH

Use the graph shown in Worked example 7.3.1 to answer the following questions.

a What is the velocity of the cyclist between E and F?	
Thinking	Working
Determine the change in position (displacement) of the cyclist between E and F using: $s = \text{final position} - \text{initial position}$	At E, $x = 300 \text{ m}$. At F, $x = 0 \text{ m}$. $s = 0 - 300$ $= -300 \text{ m}$ or 300 m backwards (that is, back towards the starting point)
Determine the time taken to travel from E to F.	$\Delta t = 100 - 80$ $= 20 \text{ s}$
Calculate the gradient of the graph between E and F using: gradient of x - t graph = $\frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = s$.	Gradient = $\frac{-300}{20}$ $= -15$
State the velocity, using: gradient of x - t graph = velocity Velocity is a vector so a direction must be given.	Since the gradient is -15 , the velocity is -15 m s^{-1} or 15 m s^{-1} backwards (towards the starting point).
b Describe the motion of the cyclist between D and E.	
Thinking	Working
Interpret the shape of the graph between D and E.	The graph is flat between D and E, indicating that the cyclist's position is not changing for this time. So the cyclist is not moving. If the cyclist is not moving, the velocity is 0 m s^{-1} .
You may confirm the result by calculating the gradient of the graph between D and E using: gradient of x - t graph = $\frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = s$.	Gradient = $\frac{0}{20}$ $= 0$

Worked example: Try yourself 7.3.2
ANALYSING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 7.3.2 to answer the following questions.

a What is the displacement of the car from 4 to 6s?	
Thinking Displacement is the area under the graph. So, calculate the area under the graph for the time period for which you want to find the displacement. Use displacement = $b \times h$ for squares and rectangles. Use displacement = $\frac{1}{2}(b \times h)$ for triangles.	Working  <p>The area from 4 to 6s is a triangle, so:</p> $\text{area} = \frac{1}{2}(b \times h)$ $= \frac{1}{2} \times 2 \times -4$ $= -4 \text{ m}$
Displacement is a vector quantity, so a direction is needed.	displacement = 4 m west
b What is the average velocity of the car from 4 to 6s?	
Thinking Identify the equation and variables, and apply the sign convention.	Working $v = \frac{s}{\Delta t}$ $s = -4 \text{ m}$ $\Delta t = 2 \text{ s}$
Substitute values into the equation: $v = \frac{s}{t}$	$v = \frac{s}{\Delta t}$ $= \frac{-4}{2}$ $= -2 \text{ m s}^{-1}$
Velocity is a vector quantity, so a direction is needed.	$v_{\text{av}} = 2 \text{ m s}^{-1} \text{ west}$

Worked example: Try yourself 7.3.3
FINDING ACCELERATION USING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 7.3.3 to answer the following question.
What is the acceleration of the car during the period from 4 to 6 s?

Thinking	Working
Acceleration is the gradient of a v - t graph. Calculate the gradient using: $\text{gradient} = \frac{\text{rise}}{\text{run}}$	Gradient from 4 to 6 = $\frac{\text{rise}}{\text{run}}$ $= \frac{-4}{2}$ $= -2 \text{ ms}^{-1}$
Acceleration is a vector quantity, so a direction is needed. Note: In this case, the car is moving in the negative direction and speeding up.	Acceleration = 2 ms^{-2} west.

Section 7.3 Review

KEY QUESTIONS SOLUTIONS

- D. The gradient is the displacement over the time taken, hence velocity.
- The car initially moves in a positive direction and travels 8 m in 2 s. It then stops for 2 s. The car then reverses direction for 5 s, passing back through its starting point after 8 s. It travels a further 2 m in a negative direction before stopping after 9 s.
- Reading from the graph:
 - +8 m
 - +8 m
 - +4 m
 - 2 m
- The car returns to its starting point when the position is zero again, which occurs at $t = 8$ s.
 - The velocity during the first 2 s is equal to the gradient of the graph during this interval.

$$\text{velocity} = \frac{\text{rise}}{\text{run}} = \frac{(0) - (8)}{2} = +4 \text{ ms}^{-1}$$
 - After 3 s the velocity is zero, since the gradient of the graph = 0.
 - velocity = gradient of graph = $\frac{\text{rise}}{\text{run}} = \frac{8 - 0}{4} = -2 \text{ ms}^{-1}$
 - The velocity at 8 s is -2 ms^{-1} , since the car is travelling at a constant velocity of -2 ms^{-1} between 4 s and 9 s.
 - The velocity from 8 s to 9 s = -2 ms^{-1} , since the car is travelling at a constant velocity of -2 ms^{-1} between 4 s and 9 s.
- Distance = $8 + 8 + 2 = 18$ m
 - Displacement = $\Delta x = (-2) - 0 = -2$ m
- Average speed = gradient of the line segment

$$= \frac{\text{rise}}{\text{run}}$$

$$= \frac{150}{30}$$

$$= 5 \text{ ms}^{-1}$$

- b** Average velocity = gradient of the line segment plus direction

$$= \frac{\text{rise}}{\text{run}}$$

$$= \frac{200}{10}$$

$$= 20 \text{ m s}^{-1} \text{ north}$$

The velocity is positive so the direction of the cyclist is north.

- c** Average velocity = displacement over time

$$= \frac{\text{rise}}{\text{run}}$$

$$= \frac{500}{50}$$

$$= 10 \text{ m s}^{-1} \text{ north}$$

- 8 a** Acceleration = gradient

$$= \frac{\text{rise}}{\text{run}}$$

$$= 0 \text{ m s}^{-2}$$

- b** Acceleration = gradient

$$= \frac{\text{rise}}{\text{run}}$$

$$= \frac{-3}{3}$$

$$= -1 \text{ or just } 1 \text{ m s}^{-2}$$

Magnitude only, so direction is not required.

- c** Split the area up into shapes and add the values together to get the full area under the graph.

Displacement = area under the graph

$$= (b \times h) + \left(\frac{1}{2} \times b \times h\right) + \left(\frac{1}{2} \times b \times h\right)$$

$$= (4 \times 1) + \left(\frac{1}{2} \times 2 \times 2\right) + \left(\frac{1}{2} \times 3 \times 3\right)$$

$$= 4 + 2 + 4.5$$

$$= 10.5 \text{ m}$$

- d** average velocity = $\frac{\text{displacement}}{\text{time}}$

$$= \frac{10.5}{7}$$

$$= 1.5 \text{ m s}^{-1}$$

- 9 a** instantaneous velocity = gradient of the line

$$= \frac{\text{rise}}{\text{run}}$$

$$= \frac{300}{15}$$

$$= 20 \text{ m s}^{-1} \text{ north}$$

- b** instantaneous velocity = gradient of the line

$$= \frac{\text{rise}}{\text{run}}$$

$$= \frac{-600}{15}$$

$$= -40 \text{ or } 40 \text{ m s}^{-1} \text{ south}$$

- 10 a** Reading from the graph, the curve flattens out after 80s.

- b** Draw a tangent to the graph at 10s and determine the gradient of the tangent.

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \text{approx. } \frac{35}{50} \text{ or } \frac{53}{40}$$

$$= 1.2 \text{ or } 1.3 \text{ m s}^{-2} \text{ (answers may vary slightly)}$$

- c Draw a tangent to the graph at 40s and determine the gradient of the tangent.

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \text{approx. } \frac{34}{90} \text{ or } \frac{35}{85}$$

$$= 0.38 \text{ or } 0.41 \text{ m s}^{-2} \text{ (answers may vary slightly)}$$

- d displacement = area under the graph

There are various methods for calculating this, but counting squares gives 49 squares, each of area 10×10 .

$$49 \times 10 \times 10 = 4900 \text{ m or } 4.9 \text{ km}$$

Section 7.4 Equations for uniform acceleration

Worked example: Try yourself 7.4.1

USING THE EQUATIONS OF MOTION

A snowboarder in a race is travelling 15 m s^{-1} east as she crosses the finishing line. She then decelerates uniformly until coming to a stop over a distance of 30 m.

a What is her acceleration as she comes to a stop?	
Thinking	Working
Write down the known quantities and the quantity that you need to find. Apply the sign convention that east is positive and west is negative.	$s = +30 \text{ m}$ $u = +15 \text{ m s}^{-1}$ $v = 0 \text{ m s}^{-1}$ $a = ?$
Identify the correct equation to use.	$v^2 = u^2 + 2as$
Substitute known values into the equation and solve for a . Include units with the answer.	$v^2 = u^2 + 2as$ $0^2 = 15^2 + 2 \times a \times 30$ $a = \frac{0 - 225}{60}$ $= -3.8 \text{ m s}^{-2}$
Use the sign convention to state the answer with its direction.	$a = 3.8 \text{ m s}^{-2}$ west
b How long does she take to come to a stop?	
Thinking	Working
Write down the known quantities and the quantity you need to find. Apply the sign convention that east is positive and west is negative.	$s = 30 \text{ m}$ $u = 15 \text{ m s}^{-1}$ $v = 0 \text{ m s}^{-1}$ $a = -3.8 \text{ m s}^{-2}$ $t = ?$
Identify the correct equation to use. Since you now know four values, any equation involving t will work.	$v = u + at$
Substitute known values into the equation and solve for t . Include units with the answer.	$t = \frac{0 - 15}{-3.8}$ $= 3.9 \text{ s}$

c What is the average velocity of the snowboarder as she comes to a stop?	
Thinking	Working
Write down the known quantities and the quantity that you need to find. Apply the sign convention that east is positive and west is negative.	$u = +15 \text{ ms}^{-1}$ $v = 0 \text{ ms}^{-1}$ $v_{\text{av}} = ?$
Identify the correct equation to use.	$v_{\text{av}} = \frac{1}{2} (u + v)$
Substitute known quantities into the equation and solve for v_{av} . Include units with the answer.	$v_{\text{av}} = \frac{1}{2} (u + v)$ $= \frac{1}{2} (15 + 0)$ $= 7.5 \text{ ms}^{-1}$
Use the sign convention to state the answer with its direction.	$v_{\text{av}} = 7.5 \text{ ms}^{-1}$ east

Section 7.4 Review

KEY QUESTIONS SOLUTIONS

- 1 E. The chosen equation must contain s , u , v and a .
- 2 a $u = 0 \text{ ms}^{-1}$, $s = 400 \text{ m}$, $t = 16 \text{ s}$, $a = ?$
 $s = ut + \frac{1}{2} at^2$
 $400 = 0 + \frac{1}{2} a \times 16^2$
 $a = \frac{400}{256} \times 2$
 $= 3.1 \text{ ms}^{-2}$
- b $u = 0 \text{ ms}^{-1}$, $s = 400 \text{ m}$, $t = 16 \text{ s}$, $a = 3.1$, $v = ?$
 $v = u + at$
 $= 0 + 3.1 \times 16$
 $= 50 \text{ ms}^{-1}$
- c $50 \text{ ms}^{-1} \times 3.6 = 180 \text{ km h}^{-1}$
- 3 a $u = 0 \text{ ms}^{-1}$, $t = 8.0 \text{ s}$, $v = 16 \text{ ms}^{-1}$, $a = ?$
 $v = u + at$
 $16 = 0 + a \times 8.0$
 $a = \frac{16}{8.0}$
 $= 2.0 \text{ ms}^{-2}$
- b $v_{\text{av}} = \frac{u + v}{2}$
 $= \frac{0 + 16}{2}$
 $= 8 \text{ ms}^{-1}$
- c $u = 0 \text{ ms}^{-1}$, $t = 8.0 \text{ s}$, $v = 16 \text{ ms}^{-1}$, $a = 2.0 \text{ ms}^{-2}$, $s = ?$
 $s = \frac{1}{2} (u + v)t$
 $= \frac{1}{2} (0 + 16) \times 8.0$
 $= 64 \text{ m}$

4 a $u = 0 \text{ ms}^{-1}$, $v = 160 \text{ ms}^{-1}$, $t = 4.0 \text{ s}$, $a = ?$

$$v = u + at$$

$$160 = 0 + a \times 4.0$$

$$a = 40 \text{ ms}^{-2}$$

b In the first 4.0s: $u = 0$, $t = 4.0$, $v = 160$, $a = 40$, $s = ?$

$$s = \frac{1}{2} (u + v)t$$

$$= \frac{1}{2} (0 + 160) \times 4.0$$

$$= 80 \times 4.0$$

$$= 320 \text{ m}$$

In the last 5.0s:

$u = 160 \text{ ms}^{-1}$, $t = 5.0 \text{ s}$, $v = 160 \text{ ms}^{-1}$, $a = 0 \text{ ms}^{-2}$, $s = ?$

$$s = \frac{1}{2} (u + v)t$$

$$= \frac{1}{2} (160 + 160) \times 5.0$$

$$= 160 \times 5.0$$

$$= 800 \text{ m}$$

Total distance in 9.0s:

$$= 320 + 800$$

$$= 1120 \text{ m}$$

$$= 1.12 \text{ km} = 1.1 \text{ km (to two significant figures)}$$

c $160 \text{ ms}^{-1} \times 3.6 = 576 \text{ km h}^{-1} = 580 \text{ km h}^{-1}$ (to two significant figures)

d $u = 0 \text{ ms}^{-1}$, $v = 160 \text{ ms}^{-1}$

$$v_{\text{av}} = \frac{u + v}{2}$$

$$= \frac{0 + 160}{2}$$

$$= 80 \text{ ms}^{-1}$$

e $v_{\text{av}} = \frac{s}{t}$

$$= \frac{1120}{9}$$

$$= 124.4 \text{ ms}^{-1} = 120 \text{ ms}^{-1}$$
 (to two significant figures)

5 a $u = 4.2 \text{ ms}^{-1}$, $t = 0.5 \text{ s}$, $v = 6.7 \text{ ms}^{-1}$, $a = ?$

$$v = u + at$$

$$6.7 = 4.2 + a \times 0.50$$

$$a = \frac{6.7 - 4.2}{0.50}$$

$$= 5.0 \text{ ms}^{-2}$$

b $u = 4.2 \text{ ms}^{-1}$, $t = 0.5 \text{ s}$, $v = 6.7 \text{ ms}^{-1}$, $a = 5.0 \text{ ms}^{-2}$, $s = ?$

$$s = \frac{1}{2} (u + v)t$$

$$= \frac{1}{2} (4.2 + 6.7) \times 0.50$$

$$= 2.725 \text{ or } 2.7 \text{ m}$$

c $v_{\text{av}} = \frac{u + v}{2}$

$$= \frac{4.2 + 6.7}{2}$$

$$= 5.45 = 5.5 \text{ ms}^{-1}$$
 (to two significant figures)

- 6** D. The stone is travelling downwards, so the velocity is downwards. As the stone strikes the water, it quickly decelerates, so the acceleration is upwards.

- 7 a** $u = -28 \text{ ms}^{-1}$, $v = 0 \text{ ms}^{-1}$, $s = -4.0 \text{ m}$, $a = ?$
 $v^2 = u^2 + 2as$
 $0 = (-28)^2 + 2 \times a \times -4.0$
 $a = \frac{-784}{-8.0}$
 $= 98 \text{ ms}^{-2}$
- b** $u = -28 \text{ ms}^{-1}$, $v = 0 \text{ ms}^{-1}$, $s = -4.0 \text{ m}$, $a = 98 \text{ ms}^{-2}$, $t = ?$
 $v = u + at$
 $0 = -28 + 98t$
 $t = \frac{28}{98}$
 $= 0.29 \text{ s}$
- c** $u = -28 \text{ ms}^{-1}$, $s = -2.0 \text{ m}$, $a = 98 \text{ ms}^{-2}$, $v = ?$
 $v^2 = u^2 + 2as$
 $= (-28)^2 + 2 \times 98 \times -2.0$
 $= 784 - 392$
 $v = 19.8 = 20 \text{ ms}^{-1}$ to two significant figures
- 8 a** $u = \frac{75 \text{ kmh}^{-1}}{3.6} = 20.83 = 21 \text{ ms}^{-1}$ to two significant figures
- b** $u = 21 \text{ ms}^{-1}$, $a = 0 \text{ ms}^{-2}$, $t = 0.25 \text{ s}$, $s = ?$
 $s = ut + \frac{1}{2} at^2$
 $= 21 \times 0.25$
 $= 5.25$
 $= 5.3 \text{ m}$ to two significant figures
- c** $u = 21$, $a = -6.0$, $v = 0$, $t = ?$
 $v^2 = u^2 + 2as$
 $0 = (21)^2 + 2 \times -6.0 \times s$
 $s = \frac{(21)^2}{12}$
 $= 36.75$
 $= 37 \text{ m}$ to two significant figures
- d** $5.3 + 37 = 42.3 = 42 \text{ m}$ to two significant figures
- 9 a** $u = 0 \text{ ms}^{-1}$, $a = 2.0 \text{ ms}^{-2}$, $s = 4.0 \text{ m}$
 $v^2 = u^2 + 2as$
 $= 0 + 2(2.0 \times 4.0)$
 $v = 4.0 \text{ ms}^{-1}$
- b** $v^2 = u^2 + 2as$
 $= 0 + 2(2.0 \times 8.0)$
 $v = 5.7 \text{ ms}^{-1}$
- c** $v = u + at$
 $4.0 = 0 + 2.0t$
 $t = 2.0 \text{ s}$
- d** $v = u + at$
 $5.7 = 0 + 2.0t$
 $t = 2.85 \text{ s}$
 The time taken to travel the final 4.0 m is $2.85 \text{ s} - 2.0 \text{ s} = 0.85 \text{ s}$.
- 10 a** $v = 12 \text{ ms}^{-1}$, $a = 1.5 \text{ ms}^{-2}$, $u = 0 \text{ ms}^{-1}$
 $v = u + at$
 $12 = 0 + 1.5t$
 $t = 8.0 \text{ s}$

- b** The bus will catch up with Anna when they have each travelled the same distance from the point at which Anna first passes the bus.

Anna: constant velocity, so $s = 12 \times t$

Bus: uniform acceleration $u = 0$, $a = 1.5 \text{ ms}^{-2}$, $s = ?$, $t = ?$

$$s = ut + \frac{1}{2} at^2$$

$$= 0.75t^2$$

When the bus catches up with Anna, their displacements are equal, so:

$$12t = 0.75t^2$$

$$t = 16 \text{ s}$$

- c** $s = 12 \times 16 = 192 \text{ m}$

Section 7.5 Vertical motion

Worked example: Try yourself 7.5.1

VERTICAL MOTION

A construction worker accidentally knocks a hammer from a building so that it falls vertically a distance of 60 m to the ground. Use $g = -9.80 \text{ ms}^{-2}$ and ignore air resistance when answering these questions.

a How long does the hammer take to fall halfway, to 30 m?	
Thinking	Working
Write down the known quantities and the quantity that you need to find. Apply the sign convention that up is positive and down is negative.	$s = -30 \text{ m}$ $u = 0 \text{ ms}^{-1}$ $a = -9.80 \text{ ms}^{-2}$ $t = ?$
Identify the correct equation for uniform acceleration to use.	$s = ut + \frac{1}{2} at^2$
Substitute known values into the equation and solve for t . Think about whether the value seems reasonable.	$-30 = 0 \times t + \frac{1}{2} \times -9.80 \times t^2$ $-30 = -4.90t^2$ $t = \sqrt{\frac{-30}{-4.90}}$ $= 2.5 \text{ s}$
b How long does it take the hammer to fall all the way to the ground?	
Thinking	Working
Write down the known quantities and the quantity that you need to find. Apply the sign convention that up is positive and down is negative.	$s = -60 \text{ m}$ $u = 0 \text{ ms}^{-1}$ $a = -9.80 \text{ ms}^{-2}$ $t = ?$
Identify the correct equation for uniform acceleration to use.	$s = ut + \frac{1}{2} at^2$
Substitute known values into the equation and solve for t . Think about whether the value seems reasonable. Notice that the hammer takes 2.5 s to travel the first 30 m and only 1.0 s to travel the final 30 m. This is because it is accelerating.	$-60 = 0 \times t + \frac{1}{2} \times -9.80 \times t^2$ $-60 = -4.90t^2$ $t = \sqrt{\frac{-60}{-4.90}}$ $= 3.5 \text{ s}$

c What is the velocity of the hammer as it hits the ground?	
Thinking	Working
Write down the known quantities and the quantity that you need to find. Apply the sign convention that up is positive and down is negative.	$s = -60 \text{ m}$ $u = 0 \text{ m s}^{-1}$ $v = ?$ $a = -9.80 \text{ m s}^{-2}$ $t = 3.5 \text{ s}$
Identify the correct equation to use. Since you now know four values, any equation involving v will work.	$v = u + at$
Substitute the known values into the equation and solve for v . Think about whether the value seems reasonable.	$v = 0 + (-9.80) \times 3.5$ $= -34 \text{ m s}^{-1}$
Use the sign and direction convention to describe the direction of the final velocity.	$v = -34 \text{ m s}^{-1}$ or 34 m s^{-1} downwards

Worked example: Try yourself 7.5.2

MAXIMUM HEIGHT PROBLEMS

On winning a cricket match, a fielder throws a cricket ball vertically into the air at 15 m s^{-1} . In the following questions, ignore air resistance and use $g = 9.80 \text{ m s}^{-2}$.

a Determine the maximum height reached by the ball.	
Thinking	Working
Write down the known quantities and the quantity that you need to find. At the maximum height the velocity is zero. Apply the sign convention that up is positive and down is negative.	$u = 15 \text{ m s}^{-1}$ $v = 0$ $a = -9.80 \text{ m s}^{-2}$ $s = ?$
Identify the correct equation to use.	$v^2 = u^2 + 2as$
Substitute known values into the equation and solve for s .	$0 = (15)^2 + 2 \times (-9.80) \times s$ $s = \frac{-225}{-19.6}$ $\therefore s = +11.5 \text{ m}$, i.e. the ball reaches a height of 11.5 m.

b Calculate the time that the ball takes to return to its starting position.	
Thinking	Working
To work out the time the ball is in the air, first calculate the time it takes to reach its maximum height. Write down the known quantities and the quantity that you need to find.	$u = 15 \text{ m s}^{-1}$ $v = 0 \text{ m s}^{-1}$ $a = -9.80 \text{ m s}^{-2}$ $s = 11.5 \text{ m}$ $t = ?$
Identify the correct equation to use.	$v = u + at$
Substitute known values into the equation and solve for t .	$0 = 15 + (-9.80 \times t)$ $9.80t = 15$ $\therefore t = 1.5 \text{ s}$ The ball takes 1.5 s to reach its maximum height. It will therefore take 1.5 s to fall from this height back to its starting point and so it takes 3.0 s to return to its starting position.

Section 7.5 Review

KEY QUESTIONS SOLUTIONS

- 1 The upwards velocity will decrease by 9.80 m s^{-1} every second until the ball stops at its highest point. The velocity will then increase by 9.80 m s^{-1} every second downwards until it hits the ground.
- 2 B. The acceleration of a falling object is due to gravity, so it is constant.
- 3 A and D. Acceleration due to gravity is constant (down), however, velocity changes throughout the journey as it is zero at the top of the flight.
- 4 **a** $u = 0 \text{ m s}^{-1}$, $a = -9.80 \text{ m s}^{-2}$, $t = 3 \text{ s}$, $v = ?$
 $v = u + at$
 $= 0 + (-9.80) \times 3.0$
 $= 29 \text{ m s}^{-1}$ (no direction required for speed)
- b** $s = -30 \text{ m}$, $u = 0 \text{ m s}^{-1}$, $a = -9.80 \text{ m s}^{-2}$, $v = ?$
 $v^2 = u^2 + 2as$
 $= 0 + 2 \times (-9.80) \times (-30)$
 $v = \sqrt{588}$
 $= 24 \text{ m s}^{-1}$
- c** $v_{\text{av}} = \frac{1}{2}(u + v)$
 $= \frac{1}{2}(0 + 24)$
 $= 12 \text{ m s}^{-1}$ (down)
- 5 **a** The same as. The acceleration of a falling object is due to gravity, so it is constant no matter the direction of vertical travel (upwards or downwards).
b The same as. The flight is symmetrical, so the starting and landing speeds are the same, but in opposite directions.
- 6 **a** $v = 0 \text{ m s}^{-1}$, $a = -9.80 \text{ m s}^{-2}$, $t = 1.5 \text{ s}$, $u = ?$
 $v = u + at$
 $0 = u - 9.80 \times 1.5$
 $u = 14.7$
 $= 15 \text{ m s}^{-1}$ (to two significant figures)
- b** $u = 15 \text{ m s}^{-1}$, $v = 0 \text{ m s}^{-1}$, $a = -9.80 \text{ m s}^{-2}$, $t = 1.5 \text{ s}$, $s = ?$
 $s = \frac{1}{2}(u + v)t$
 $= \frac{1}{2}(15 + 0) \times 1.5$
 $= 11.25$
 $= 11 \text{ m}$ (to two significant figures)
- 7 **a** $u = 0 \text{ m s}^{-1}$, $a = -9.80 \text{ m s}^{-2}$, $t = 0.40 \text{ s}$, $v = ?$
 $v = u + at$
 $= 0 - 9.80 \times 0.40$
 $= -3.92$
 $= -3.9 \text{ m s}^{-1}$ (to two significant figures)
- b** $u = 0 \text{ m s}^{-1}$, $a = -9.80 \text{ m s}^{-2}$, $t = 0.40 \text{ s}$, $v = -3.9 \text{ m s}^{-1}$, $s = ?$
 $s = \frac{1}{2}(u + v)t$
 $= \frac{1}{2}(0 + 3.9) \times 0.40$
 $= -0.78$
 $= 0.78 \text{ m}$

c $u = 0, a = -9.80, t = 0.20, s = ?$

$$s = ut + \frac{1}{2} at^2$$

$$= 0 + \frac{1}{2} \times -9.80 \times (0.20)^2$$

$$= -0.20$$

$$= 0.20 \text{ m}$$

d Distance in last 0.20 s = $0.78 - 0.20$
 $= 0.58 \text{ m}$

8 a The time to the top is half of the total time, i.e. 2.0s.

b $v = 0 \text{ ms}^{-1}, a = -9.80 \text{ ms}^{-2}, t = 2 \text{ s}, u = ?$

$$v = u + at$$

$$0 = u + -9.80 \times 2$$

$$u = 9.80 \times 2$$

$$= 19.6 \text{ or } 20 \text{ ms}^{-1}$$

c $v = 0, a = -9.80, t = 2, u = 19.6, s = ?$

$$s = vt + \frac{1}{2} at^2$$

$$= 0 + \frac{1}{2} \times -9.80 \times (2)^2$$

$$= 19.6$$

$$= 20 \text{ m (to two significant figures)}$$

d The lid returns to its starting position, so the final velocity will be same as the launch velocity, but in the opposite direction, i.e. 20 ms^{-1} downwards.

9 a Shot-put: $u = 0 \text{ ms}^{-1}, a = -9.80 \text{ ms}^{-2}, s = -60.0 \text{ m}, t = ?$

$$s = ut + \frac{1}{2} at^2$$

$$-60.0 = 0 + \frac{1}{2} \times -9.80 \times t^2$$

$$t^2 = \frac{60}{\frac{1}{2} \times 9.80}$$

$$t = 3.5 \text{ s}$$

b 100 g mass: $u = -10.0 \text{ ms}^{-1}, a = -9.80 \text{ ms}^{-2}, s = -70.0 \text{ m}, v = ?, t = ?$

$$v^2 = u^2 + 2as$$

$$= (-10.0)^2 + 2 \times (-9.80) \times (-70.0)$$

$$= 1472$$

$$v = \pm 38.4 \text{ ms}^{-1}$$

Because the mass has a downwards velocity, we use the negative value.

$$v = -38.4 \text{ ms}^{-1}$$

$$v = u + at$$

$$-38.4 = -10.0 - 9.80t$$

$$9.80t = -10.0 + 38.4$$

$$t = 2.9 \text{ s}$$

You can also solve this using the formula $s = ut + \frac{1}{2} at^2$ and the quadratic formula.

10 a $s = vt - \frac{1}{2} at^2$

$$15.0 = 0 - 0.5 \times 9.80 \times t^2$$

$$t = 1.7 \text{ s}$$

b From maximum height of 15.0m, the ball will fall by 11.0m. Find how long it takes to travel this 11.0m.

$$s = ut - \frac{1}{2} at^2$$

$$-11.0 = 0 + 0.5 \times (-9.80) \times t^2$$

$$t = 1.5 \text{ s}$$

$$\text{Total time from bounce} = 1.7 + 1.5 = 3.2 \text{ s}$$

11 a $u = 8.00 \text{ ms}^{-1}$, $a = -9.80 \text{ ms}^{-2}$, $t = 3.00 \text{ s}$

It is at its maximum height when $v = 0$.

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a}$$

$$= \frac{0 - 8.00^2}{2 \times (-9.80)}$$

$$= 3.27 \text{ m}$$

b when $v = 0$

$$v = u + at$$

$$t = \frac{v - u}{a} = \frac{0 - 8.00}{-9.80}$$

$$= 0.816 \text{ s}$$

c time taken to fall = 3.00 s

$$s = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$= 8.00 \times 3.00 + \frac{1}{2} \times (-9.80) \times (3.00)^2$$

$$s = -20.1 \text{ m}$$

height of cliff is 20.1 m above the sea

CHAPTER 7 REVIEW

1 $\frac{95}{3.6} = 26 \text{ ms}^{-1}$

2 $15 \times 3.6 = 54 \text{ km h}^{-1}$

3 average speed = $\frac{\text{distance}}{\text{time}}$

$$= \frac{15 + 5 + 5 + 5}{3.0}$$

$$= 10 \text{ km h}^{-1}$$

4 a average velocity = $\frac{\text{displacement}}{\text{time}}$

$$= \frac{20}{3.0}$$

$$= 6.7 \text{ km h}^{-1} \text{ north}$$

b $6.7 \text{ km h}^{-1} \text{ north} = \frac{6.7}{3.6}$

$$= 1.9 \text{ ms}^{-1} \text{ north}$$

5 $\Delta v = 4.0 - 6.0$

$$= -2.0 \text{ ms}^{-1}$$

The change in speed is -2.0 ms^{-1} . That is, it has decreased by 2.0 ms^{-1} . Speed is a scalar and has no direction.

6 B. The car is moving in a positive direction so its velocity is positive. The car is slowing down so its acceleration is negative.

7 $a = \frac{v - u}{t}$

$$= \frac{-15}{2.5}$$

$$= -6 \text{ ms}^{-2}$$

or

$$u = 15 \text{ ms}^{-1}, v = 0 \text{ ms}^{-1}, t = 2.5 \text{ s}, a = ?$$

$$v = u + at$$

$$0 = 15 + a \times 2.5$$

$$a = \frac{-15}{2.5}$$

$$= -6 \text{ ms}^{-2}$$

- 8**
- a** The only positive gradient section is from 10 to 25 s.
 - b** The only negative gradient section is from 30 to 45 s.
 - c** The motorbike is stationary when the sections on the position–time graph are horizontal. The horizontal sections are from 0 to 10 s, from 25 to 30 s and from 45 to 60 s.
 - d** The zero position is at 42.5 s or 43 s.
- 9**
- a** Graph B is the correct answer as it shows speed decreasing to zero to show the car stopping.
 - b** Graph A is the correct graph because it shows a constant value for speed. This is indicated by a straight horizontal line on a velocity–time graph.
 - c** Graph C is the correct graph because it shows velocity increasing from zero in a straight line, indicating uniform acceleration.
- 10**
- a** Displacement is the area under a velocity–time graph. Area can be determined by counting squares under the graph, then multiplying by the area of each square. This gives approximately 57 squares $\times (2 \times 1) = 114$ m. Alternatively, you can break the area into various shapes and find the sum of their areas:
 $72 + 14 + 18 + 10 = 114$ m.
 The result is positive, which means the displacement is north of the starting point.
 The cyclist's displacement is 114 m north.

b Average velocity = $\frac{\text{displacement}}{\text{time}}$

$$= \frac{114}{11.0}$$

$$= 10.4 \text{ m s}^{-1}$$

- c** Acceleration is the gradient of the graph. At $t = 1$ s, the gradient is flat and therefore zero. This could also be calculated as follows:

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

$$= 0 \text{ m s}^{-2}$$

- d** Acceleration at $t = 10$ s is:

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

$$= -\frac{14}{2}$$

$$= -7 \text{ or } 7 \text{ m s}^{-2} \text{ south}$$

- e** A. The velocity is always positive (or zero) indicating that the cyclist only travelled in one direction.

- 11** $u = 0 \text{ m s}^{-1}$, $a = 3.5 \text{ m s}^{-2}$, $t = 4.5$ s, $v = ?$

$$v = u + at$$

$$= 0 + 3.5 \times 4.5$$

$$= 15.75$$

$$= 16 \text{ m s}^{-1} \text{ (to two significant figures)}$$

- 12** **a** $u = 0 \text{ m s}^{-1}$, $s = 2$ m, $t = 1$ s, $a = ?$

$$s = ut + \frac{1}{2} at^2$$

$$2.0 = 0 + \frac{1}{2} \times a \times (1.0)^2$$

$$a = 4.0 \text{ m s}^{-2}$$

- b** $u = 0 \text{ m s}^{-1}$, $t = 1$ s, $a = 4 \text{ m s}^{-2}$, $v = ?$

$$v = u + at$$

$$= 0 + 4.0 \times 1.0$$

$$= 4.0 \text{ m s}^{-1}$$

- c** After 2.0 s the total distance travelled:

$$u = 0 \text{ m s}^{-1}$$
, $t = 2$ s, $a = 4 \text{ m s}^{-2}$, $s = ?$

$$s = ut + \frac{1}{2} at^2$$

$$= 0 + 0.5 \times 4.0 \times (2.0)^2$$

$$= 8.0 \text{ m}$$

Distance travelled during the 2nd second = $8.0 - 2.0 = 6.0$ m.

13 a $u = 10 \text{ ms}^{-1}$, $v = 0 \text{ ms}^{-1}$, $s = 10 \text{ m}$, $a = ?$

$$v^2 = u^2 + 2as$$

$$0 = 10^2 + 2 \times a \times 10$$

$$a = -\frac{100}{20}$$

$$= -5.0 \text{ ms}^{-2}$$

b $u = 10 \text{ ms}^{-1}$, $v = 0 \text{ ms}^{-1}$, $s = 10 \text{ m}$, $a = -5 \text{ ms}^{-2}$, $t = ?$

$$v = u + at$$

$$0 = 10 - 5t$$

$$t = 2.0 \text{ s}$$

14 a She starts at + 4 m.

b She is at rest during section A and C.

c She is moving in a positive direction during section B with a velocity $+0.8 \text{ ms}^{-1}$.

d She is moving in the negative direction at 2.4 ms^{-1} during section D.

e $d = 8 + 12$

$$= 20 \text{ m}$$

$$t = 25 \text{ s}$$

$$v_{\text{av}} = \frac{s}{t}$$

$$= \frac{20}{25}$$

$$= 0.8 \text{ ms}^{-1}$$

15 a acceleration = gradient

$$= \frac{\text{rise}}{\text{run}}$$

$$= \frac{8}{4}$$

$$= 2 \text{ ms}^{-1}$$

b The bus will overtake the bike when they have both travelled the same distance, given by the areas under the two graphs. After 8 s, the bus has travelled 56 m and the bike 64 m. After 10 s, the bus has travelled 80 m and the bike 80 m.

Algebraically, this could be determined by:

The displacement for the bus = $56 + 12(t - 8)$

The displacement for the bike = $8t$

Equating these two displacements gives:

$$8t = 56 + 12t - 96$$

$$12t - 8t = 96 - 56$$

$$4t = 40$$

$$t = 10 \text{ s}$$

c After 10 s the bike has travelled $10 \times 8 = 80 \text{ m}$.

d average velocity $v_{\text{av}} = \frac{\text{displacement}}{\text{time taken}}$

$$\text{Displacement} = \left(\frac{1}{2} \times 4 \times 8\right) + (4 \times 8) + \left(\frac{1}{2} \times 4 \times 4\right)$$

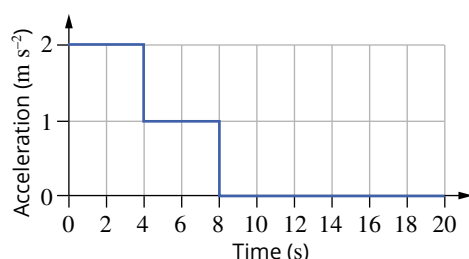
$$= 16 + 32 + 8$$

$$= 56 \text{ m}$$

$$\text{So } v_{\text{av}} = \frac{56}{8}$$

$$= 7 \text{ ms}^{-1}$$

16 a



- b** The change in velocity of the bus over the first 8 s is determined by calculating the area under the acceleration–time graph from $t = 0$ to $t = 8$ s, i.e. $+12 \text{ m s}^{-1}$.
- 17** The marble slows down by 9.80 m s^{-1} each second so it will take 4 s to stop momentarily at the top of its motion. It has a positive velocity that changes to zero on the way up. Its acceleration is constant at -9.80 m s^{-2} due to gravity.
- 18** D. The acceleration of a falling object is due to gravity, so it is constant.
- 19** B. Initial velocity is upwards, it is zero at the top and downwards on the way back down. Acceleration due to gravity is always downwards.
- 20 a** The area under the v – t graph up to 3 s gives:

$$s = \frac{1}{2} \times 3 \times 30$$

$$= 45 \text{ m}$$

or

$$u = 30, v = 0, t = 3, s = ?$$

$$s = \frac{1}{2} (u + v)t$$

$$= \frac{1}{2} (30 + 0) \times 3$$

$$= 45 \text{ m}$$

- b** From the graph, the ball goes up for 3 s then down for 3 s, giving a total time of 6 s, or:

$$u = 30 \text{ m s}^{-1}, v = -30 \text{ m s}^{-1}, a = -10 \text{ m s}^{-2}, t = ?$$

$$v = u + at$$

$$-30 = 30 - 10t$$

$$t = \frac{60}{10}$$

$$= 6 \text{ s}$$

- c** From the v – t graph, the velocity at $t = 5$ s is -20 or 20 m s^{-1} down, or:

$$u = 30 \text{ m s}^{-1}, a = -10 \text{ m s}^{-2}, t = 5 \text{ s}, v = ?$$

$$v = u + at$$

$$= 30 + (-10) \times 5$$

$$= 30 - 50$$

$$= -20 \text{ m s}^{-1}$$

A negative value indicates down, therefore the correct answer is 20 m s^{-1} down.

- d** Acceleration is always 10 m s^{-2} down.

- 21 a** Balloon: $u = -8.0 \text{ m s}^{-1}, a = 0 \text{ m s}^{-2}, s = -80 \text{ m}, t = ?$

The balloon has constant speed. Use $v_{av} = \frac{s}{t}$ so:

$$t = \frac{s}{v_{av}}$$

$$= \frac{80}{8.0}$$

$$= 10 \text{ s}$$

- b** Coin: $u = -8.0 \text{ m s}^{-1}, a = -9.80 \text{ m s}^{-2}, s = -80 \text{ m}, v = ?$

$$v^2 = u^2 + 2as$$

$$= (-8)^2 + 2 \times -9.80 \times -80$$

$$= 64 + 1568$$

$$v = 40.4$$

$$= 40 \text{ m s}^{-1} \text{ to two significant figures}$$

- c** Coin: $u = -8.0, a = -9.80, s = -80, v = -40.4, t = ?$

$$v = u + at$$

$$-40.4 = -8.0 - 9.80t$$

$$9.80t = -8.0 + 40.4$$

$$t = 3.3 \text{ s}$$

Balloon takes 10 s to land, coin takes 3.3 s, so $10 - 3.3 = 6.7$ s difference.

22 $t = 1.5\text{ s}$, $v = 0\text{ ms}^{-1}$, $a = -9.80\text{ ms}^{-2}$, $u = ?$

$$v = u + at$$

$$0 = u + (-9.80 \times 1.5)$$

$$u = 14.7$$

$$u = 15\text{ ms}^{-1} \text{ up}$$

23 $t = 1.5\text{ s}$, $v = 0\text{ ms}^{-1}$, $a = -9.80\text{ ms}^{-2}$, $u = 14.7\text{ ms}^{-1}$, $s = ?$

$$v^2 = u^2 + 2as$$

$$0 = (14.7)^2 + 2 \times -9.80 \times s$$

$$s = 11\text{ m}$$

Chapter 8 Momentum and forces

Section 8.1 Momentum and conservation of momentum

Worked example: Try yourself 8.1.1

MOMENTUM

Calculate the momentum of a 1230 kg car driving at 16.7 ms^{-1} north.	
Thinking	Working
Ensure that the variables are in their standard units.	$m = 1230 \text{ kg}$ $v = 16.7 \text{ ms}^{-1}$ north
Apply the equation for momentum.	$p = mv$ $= 1230 \times 16.7$ $= 20541$ $= 20500 \text{ kgms}^{-1}$
Ensure that the final answer is in the same direction as the velocity.	$p = 20500 \text{ kgms}^{-1}$ north

Worked example: Try yourself 8.1.2

CONSERVATION OF MOMENTUM

A 1200 kg wrecking ball is moving at 2.50 ms^{-1} north towards a 1500 kg wrecking ball moving at 4.00 ms^{-1} south. Calculate the final velocity of the 1500 kg ball if the 1200 kg ball rebounds at 3.50 ms^{-1} south.	
Thinking	Working
Identify the variables using subscripts. Ensure that the variables are in their standard units.	$m_1 = 1200 \text{ kg}$ $u_1 = 2.50 \text{ ms}^{-1}$ north $v_1 = 3.50 \text{ ms}^{-1}$ south $m_2 = 1500 \text{ kg}$ $u_2 = 4.00 \text{ ms}^{-1}$ south $v_2 = ?$
Apply the sign convention to the variables.	$m_1 = 1200 \text{ kg}$ $u_1 = +2.50 \text{ ms}^{-1}$ $v_1 = -3.50 \text{ ms}^{-1}$ $m_2 = 1500 \text{ kg}$ $u_2 = -4.00 \text{ ms}^{-1}$ $v_2 = ?$
Apply the equation for conservation of momentum.	$\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$ $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ $(1200 \times 2.50) + (1500 \times -4.00) = (1200 \times -3.50) + 1500 v_2$ $1500 v_2 = 3000 + -6000 - (-4200)$ $v_2 = \frac{1200}{1500}$ $= 0.800 \text{ ms}^{-1}$
Apply the sign convention to describe the direction of the final velocity.	$v_2 = 0.80 \text{ ms}^{-1}$ north

Worked example: Try yourself 8.1.3
CONSERVATION OF MOMENTUM WHEN MASSES COMBINE

An 80.0 kg rugby player is moving at 1.50 m s^{-1} north when he tackles an opponent with a mass of 50.0 kg who is moving at 5.00 m s^{-1} south. Calculate the final velocity of the two players.	
Thinking	Working
Identify the variables using subscripts and ensure that the variables are in their standard units. Add m_1 and m_2 to get m_3 .	$m_1 = 80.0 \text{ kg}$ $u_1 = 1.50 \text{ m s}^{-1}$ north $m_2 = 50.0 \text{ kg}$ $u_2 = 5.00 \text{ m s}^{-1}$ south $m_3 = 130 \text{ kg}$ $v_3 = ?$
Apply the sign convention to the variables.	$m_1 = 80.0 \text{ kg}$ $u_1 = +1.50 \text{ m s}^{-1}$ $m_2 = 50.0 \text{ kg}$ $u_2 = -5.00 \text{ m s}^{-1}$ $m_3 = 130 \text{ kg}$ $v_3 = ?$
Apply the equation for conservation of momentum.	$\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$ $m_1 u_1 + m_2 u_2 = m_3 v_3$ $(80 \times 1.50) + (50 \times -5.00) = 130 v_3$ $v_3 = \frac{120 + -250}{130}$ $= -1.00 \text{ m s}^{-1}$
Apply the sign to describe the direction of the final velocity.	$v_3 = 1.00 \text{ m s}^{-1}$ south

Worked example: Try yourself 8.1.4
CONSERVATION OF MOMENTUM FOR EXPLOSIVE COLLISIONS

A 2000 kg cannon fires a 10 kg cannonball. The cannon and the cannonball are initially stationary. After firing, the cannon recoils with a velocity of 8.15 m s^{-1} north. Calculate the velocity of the cannonball just after it is fired.	
Thinking	Working
Identify the variables using subscripts and ensure that the variables are in their standard units. Note that m_1 is the sum of the bodies i.e. the cannon and the cannonball.	$m_1 = 2010 \text{ kg}$ $u_1 = 0 \text{ m s}^{-1}$ $m_2 = 2000 \text{ kg}$ $u_2 = 8.15 \text{ m s}^{-1}$ north $m_3 = 10 \text{ kg}$ $v_3 = ?$
Apply the sign convention to the variables.	$m_1 = 2010 \text{ kg}$ $u_1 = 0 \text{ m s}^{-1}$ $m_2 = 2000 \text{ kg}$ $u_2 = +8.15 \text{ m s}^{-1}$ $m_3 = 10 \text{ kg}$ $v_3 = ?$

Apply the equation for conservation of momentum for explosive collisions.	$\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$ $m_1 u_1 = m_2 u_2 + m_3 v_3$ $2010 \times 0 = (2000 \times 8.15) + 10v_3$ $v_3 = \frac{0 - 16300}{10}$ $\frac{-16300}{10}$ $= -1630 \text{ ms}^{-1}$
Apply the sign to describe the direction of the final velocity.	$v_3 = 1630 \text{ ms}^{-1} \text{ south}$

Section 8.1 Review

KEY QUESTIONS SOLUTIONS

1 $p = mv$
 $= 3.50 \times 2.50$
 $= 8.75 \text{ kg ms}^{-1} \text{ south}$

2 $p = mv$
 $= 433 \times 22.2$
 $= 9612.6$
 $= 9610 \text{ kg ms}^{-1} \text{ west}$

3 $p = mv$
 $= 0.065 \times 61.0$
 $= 3.97 \text{ kg ms}^{-1} \text{ south}$

4 First ball: $p = mv = 4.5 \times 3.5 = 15.75 \text{ kg ms}^{-1}$
 Second ball: $p = mv = 2.5 \times 6.8 = 17 \text{ kg ms}^{-1}$
 The second ball has the greater momentum.

5
$$\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(70.0 \times 0) + (400 \times 0) = (70.0 \times 2.50) + 400v_2$$

$$400v_2 = 0 + -175$$

$$v_2 = \frac{-175}{400}$$

$$= -0.438 \text{ ms}^{-1}$$

The boat moves backwards at 0.438 ms^{-1} (to 3 significant figures).

6
$$\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(0.070 \times 0) + (0.545 \times 80) = (0.070 \times 75.0) + 0.545v_2$$

$$0.545v_2 = 0 + 43.6 - 5.25$$

$$v_2 = \frac{38.35}{0.545}$$

$$= 70.4 \text{ ms}^{-1} \text{ (to 3 significant figures)}$$

7
$$\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$$

$$m_1 u_1 + m_2 u_2 = m_3 v_3$$

$$(2500 \times 2.00) + (m_2 \times 0) = (2500 + m_2) \times 0.300$$

$$5000 = 0.300 \times 2500 + 0.300m_2$$

$$0.300m_2 = 5000 - 750$$

$$m_2 = \frac{4250}{0.300}$$

$$= 14167$$

$$= 14200 \text{ kg (to 3 significant figures)}$$

- 8 $\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$
 $m_1 u_1 = m_2 v_2 + m_3 v_3$
 $0 = (9995)v_2 + (5.0 \times 6000)$
 $v_2 = \text{velocity of space shuttle} = 3.0 \text{ ms}^{-1}$ in the direction opposite to that of the exhaust gases.
- 9 a $F_{\text{gas}} = ma = m \times \frac{(v - u)}{t}$
 $= 50 \times \left(\frac{180 - 0}{2} \right)$
 $= 4.5 \times 10^3 \text{ N downwards}$
- b The force of the exhaust gas on the rocket is equal and opposite to the force of the rocket on the exhaust gas.
 $F_{\text{rocket}} = 4.5 \times 10^3 \text{ N upwards}$
- c Net upwards acceleration = $\frac{\text{resultant force}}{\text{mass}}$
 $= \frac{4.5 \times 10^3 - (225 \times 9.80)}{225}$
 $= 10.2 \text{ ms}^{-2}$
- d $v = u + at$
 $= (0) + (10.2) \times (2)$
 $= 20.4 \text{ ms}^{-1}$ upwards

Section 8.2 Change in momentum and impulse

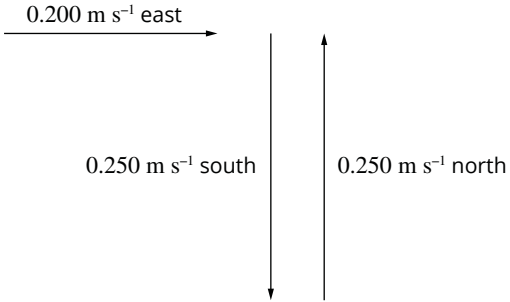
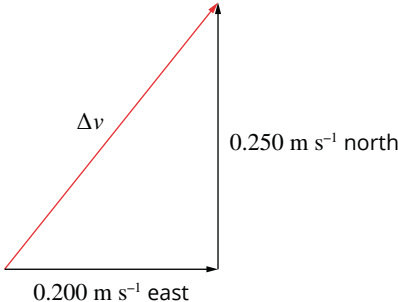
Worked example: Try yourself 8.2.1

IMPULSE OR CHANGE IN MOMENTUM

A student hurries to class after lunch, moving at 4.55 ms^{-1} north. Suddenly the student remembers that she has forgotten her laptop and goes back to her locker at 6.15 ms^{-1} south. If her mass is 75.0 kg , calculate the impulse of the student during the time it takes to turn around.	
Thinking	Working
Ensure that the variables are in their standard units.	$m = 75.0 \text{ kg}$ $u = 4.55 \text{ ms}^{-1}$ north $v = 6.15 \text{ ms}^{-1}$ south
Apply the sign convention to the velocity vectors.	$m = 75.0 \text{ kg}$ $u = 4.55 \text{ ms}^{-1}$ $v = -6.15 \text{ ms}^{-1}$
Apply the equation for impulse or change in momentum.	$I = mv - mu$ $= (75.0 \times -6.15) - (75.0 \times 4.55)$ $= -461.25 - 341.25$ $= -802.5 \text{ kg ms}^{-1}$ $= -803 \text{ kg ms}^{-1}$
Apply the sign convention to describe the direction of the impulse.	$I = 803 \text{ kg ms}^{-1}$ south

Worked example: Try yourself 8.2.2
IMPULSE OR CHANGE IN MOMENTUM IN TWO DIMENSIONS

A 65.0 g pool ball is moving at 0.250 m s^{-1} south towards a cushion and bounces off at 0.200 m s^{-1} east. Calculate the impulse on the ball during the change in velocity.

Thinking	Working
Identify the formula for calculating a change in velocity, Δv .	$\Delta v = \text{final velocity} - \text{initial velocity}$
Draw the final velocity, v , and the initial velocity, u , separately. Then draw the initial velocity in the opposite direction, which represents the negative of the initial velocity, $-u$.	
Construct a vector diagram, drawing first v and then from its head draw the opposite of u . The change of velocity vector is drawn from the tail of the final velocity to the head of the opposite of the initial velocity.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the change in velocity.	$\Delta v^2 = 0.2^2 + 0.25^2$ $= 0.0400 + 0.0625$ $\Delta v = \sqrt{0.1025}$ $= 0.320 \text{ m s}^{-1}$
Calculate the angle from the north vector to the change in velocity vector.	$\tan \theta = \frac{0.200}{0.250}$ $\theta = \tan^{-1} 0.800$ $= 38.7^\circ$
State the magnitude and direction of the change in velocity.	$\Delta v = 0.320 \text{ m s}^{-1} \text{ N } 38.7^\circ \text{ E}$
Identify the variables using subscripts and ensure that the variables are in their standard units.	$m_1 = 0.0650 \text{ kg}$ $\Delta v = 0.320 \text{ m s}^{-1} \text{ N } 38.7^\circ \text{ E}$
Apply the equation for impulse or change in momentum.	$\Delta p = mv - mu$ $= m(v - u)$ $= m\Delta v$ $= 0.065 \times 0.320$ $= 0.0208 \text{ kg m s}^{-1}$
Apply the direction convention to describe the direction of the change in momentum.	$\Delta p = 0.0208 \text{ kg m s}^{-1} \text{ N } 38.7^\circ \text{ E}$

Section 8.2 Review

KEY QUESTIONS SOLUTIONS

- 1 $I = mv - mu$
 $= (9.50 \times -6.25) - (9.50 \times 2.50)$
 $= -59.375 - 23.75$
 $= -83.1 \text{ kg m s}^{-1}$
 $= 83.1 \text{ kg m s}^{-1}$ south
- 2 $I = mv - mu$
 $= (6050 \times 16.7) - (6050 \times -22.2)$
 $= 101\,035 + 134\,310$
 $= 235\,000 \text{ kg m s}^{-1}$ east
- 3 $\Delta p = mv - mu$
 $= (8.00 \times 8.00) - (8.00 \times 3.00)$
 $= 64.0 - 24.0$
 $= 40.0 \text{ kg m s}^{-1}$ east
- 4 $\Delta p = mv - mu$
 $= (0.250 \times -9.80) - (0.250 \times 0)$
 $= -2.45 \text{ kg m s}^{-1}$
 $= 2.45 \text{ kg m s}^{-1}$ down
- 5 $\Delta p = mv - mu$
 $mv = \Delta p + mu$
 $v = \frac{\Delta p + mu}{m}$
 $= \frac{-0.075 + 0.125 \times 3.00}{0.125}$
 $= 2.4 \text{ m s}^{-1}$ north
- 6 $\Delta v = \text{final velocity} - \text{initial velocity}$
 $= v - u$
 $= v + (-u)$
 $= 45.0 \text{ m s}^{-1}$ north + 45.0 m s^{-1} east

The magnitude of the change in velocity is calculated using Pythagoras' theorem:

$$\Delta v^2 = 45.0^2 + 45.0^2$$

$$= 2025 + 2025$$

$$\Delta v = \sqrt{4050}$$

$$= 63.6 \text{ m s}^{-1}$$

Use trigonometry to calculate the angle of the change in momentum.

$$\tan \theta = \frac{45.0}{45.0}$$

$$\theta = \tan^{-1}(1)$$

$$= 45^\circ$$

$$\Delta v = 63.6 \text{ m s}^{-1} \text{ N } 45^\circ \text{ E}$$

The magnitude of the change in momentum is calculated using:

$$\Delta p = mv - mu$$

$$= m(v - u)$$

$$= m\Delta v$$

$$= 45.0 \times 63.6$$

$$= 2862$$

$$= 2860 \text{ kg m s}^{-1}$$

$$\Delta p = 2860 \text{ kg m s}^{-1} \text{ N } 45^\circ \text{ E}$$

$$\begin{aligned}
 7 \quad \Delta v &= \text{final velocity} - \text{initial velocity} \\
 &= v - u \\
 &= v + (-u) \\
 &= 3.60 \text{ m s}^{-1} \text{ west} + 4.00 \text{ m s}^{-1} \text{ south}
 \end{aligned}$$

The magnitude of the change in velocity is calculated using Pythagoras' theorem:

$$\begin{aligned}
 \Delta v^2 &= 3.60^2 + 4.00^2 \\
 &= 12.96 + 16.0 \\
 \Delta v &= \sqrt{28.9} \\
 &= 5.38 \text{ m s}^{-1}
 \end{aligned}$$

Use trigonometry to calculate the angle of the change in momentum.

$$\begin{aligned}
 \tan \theta &= \frac{3.60}{4.00} \\
 \theta &= \tan^{-1}(0.9) \\
 &= 42^\circ
 \end{aligned}$$

$$\Delta v = 5.38 \text{ m s}^{-1} \text{ S } 42^\circ \text{ W}$$

The magnitude of the change in momentum is calculated using:

$$\begin{aligned}
 \Delta p &= mv - mu \\
 &= m(v - u) \\
 &= m\Delta v \\
 &= 70.0 \times 5.38 \\
 &= 377 \text{ kg m s}^{-1}
 \end{aligned}$$

$$\Delta p = 377 \text{ kg m s}^{-1} \text{ S } 42^\circ \text{ W}$$

Section 8.3 Newton's first law

Section 8.3 Review

KEY QUESTIONS SOLUTIONS

- The box has changed its velocity so the student can use Newton's first law to conclude that an unbalanced force must have acted on the box to slow it down.
- Even though the car has maintained its speed, the direction has changed, which means the velocity has changed. Using Newton's first law, it can be concluded that an unbalanced force has acted on the car to change its direction.
- B. Since the ball maintains a constant velocity, according to Newton's first law there must not be an unbalanced force. There is no forwards force, friction or air resistance acting on the ball.
- No horizontal force acts on the person. In accordance with Newton's first law of motion, the bus slows, but the standing passenger will continue to move with constant velocity unless acted on by an unbalanced force; usually the passenger will lose his or her footing and fall forwards.
- Constant velocity, so $F_{\text{net}} = 0$, then frictional force = applied force = 20 N.
- Constant velocity, so $F_{\text{net}} = 0$, then frictional force = applied force = 25 N.
 - 25 N
 - $F \cos 30^\circ = 25 \text{ N}$
 $F = 29 \text{ N}$ at an angle of 30° to the horizontal.
- The plane slows down as it travels along the runway because of the large retarding forces acting on it. The passengers wearing seatbelts would have retarding forces provided by the seatbelt and would slow down at the same rate as the plane. A passenger standing in the aisle, if they were not hanging on to anything, would have no retarding forces acting and so would tend to maintain their original velocity and move towards the front of the plane.
- Gravitational force of attraction between the two masses.
 - Electrical force of attraction between the negative electron and the positive nucleus.
 - Friction between the tyres and the road.
 - Tension in the wire.

- 9 a If the cloth is pulled quickly, the force on the glass acts for a short time only. This force does not overcome the tendency of the glass to stay where it is, i.e. its inertia.
 b Using a full glass makes the trick easier because the force will have less effect on the glass due to its greater mass. The inertia of the full glass is greater than that of an empty glass.
- 10 The fully laden semitrailer will find it most difficult to stop. Its large mass means that more force is required to bring it to a stop.
- 11 Constant speed, so $F_{\text{net}} = 0$ in both vertical and horizontal directions. To exactly balance the other forces, lift = 50 kN up, and drag = 12 kN west.

Section 8.4 Newton's second law

Worked example: Try yourself 8.4.1

CALCULATING THE FORCE THAT CAUSES AN ACCELERATION

Calculate the net force causing a 75.8 kg runner to accelerate at 4.05 m s^{-2} south.	
Thinking	Working
Ensure that the variables are in their standard units.	$m = 75.8 \text{ kg}$ $a = 4.05 \text{ m s}^{-2}$ south
Apply the equation for force from Newton's second law.	$F_{\text{net}} = ma$ $= 75.8 \times 4.05$ $= 307 \text{ N}$
Give the direction of the net force, which is the same as the direction of the acceleration.	$F_{\text{net}} = 307 \text{ N south}$

Worked example: Try yourself 8.4.2

CALCULATING THE FINAL VELOCITY OF AN ACCELERATING MASS

Calculate the final velocity of a 307 g fish that accelerates for 5.20 s from rest due to a force of 0.250 N left.	
Thinking	Working
Ensure that the variables are in their standard units.	$m = 0.307 \text{ kg}$ $t = 5.20 \text{ s}$ $u = 0 \text{ m s}^{-1}$ $F_{\text{net}} = 0.250 \text{ N left}$
Apply a variations of the equation for force from Newton's second law.	$F_{\text{net}} = m \frac{(v - u)}{t}$ $(v - u) = \frac{(F_{\text{net}} t)}{m}$ $v = \frac{(F_{\text{net}} t)}{m} + u$ $= \frac{0.250 \times 5.20}{0.307} + 0$ $= 4.23 \text{ m s}^{-1}$
Give the direction of the final velocity as being the same as the direction of the force.	$v = 4.23 \text{ m s}^{-1}$ left

Worked example: Try yourself 8.4.3
CALCULATING THE ACCELERATION OF AN OBJECT WITH MORE THAN ONE FORCE ACTING ON IT

A car with a mass of 900 kg applies a driving force of 3000 N as it starts moving. Friction and air resistance oppose the motion of the car with a force of 750 N. What is the car's initial acceleration?	
Thinking	Working
Determine the individual forces acting on the car, and apply the vector sign convention.	$F_1 = 3000 \text{ N forwards}$ $= 3000 \text{ N}$ $F_2 = 750 \text{ N backwards}$ $= -750 \text{ N}$
Determine the net force acting on the car.	$F_{\text{net}} = F_1 + F_2$ $= 3000 + (-750)$ $= +2250 \text{ N or } 2250 \text{ N forwards}$
Use Newton's second law to determine the acceleration.	$a = \frac{F_{\text{net}}}{m}$ $= \frac{2250}{900}$ $= 2.50 \text{ m s}^{-2} \text{ forwards}$

Worked example: Try yourself 8.4.4
CALCULATING THE ACCELERATION OF A CONNECTED BODY

A 0.6 kg trolley cart is connected by a cord to a 1.5 kg mass. The cord is placed over a pulley and the mass is allowed to fall under the influence of gravity.

a Assuming that the cart can move over the table unhindered by friction, determine the acceleration of the cart.	
Thinking	Working
Recognise that the cart and the falling mass are connected, and determine a sign convention for the motion.	As the mass falls, the cart will move forwards. Therefore, both downwards movement of the mass and forwards movement of the cart will be considered positive motion.
Write down the data that is given. Apply the sign convention to vectors.	$m_1 = 1.5 \text{ kg}$ $m_2 = 0.6 \text{ kg}$ $g = 9.8 \text{ m s}^{-2} \text{ down}$ $= +9.8 \text{ m s}^{-2}$
Determine the forces acting on the system.	The only force acting on the combined system of the cart and mass is the weight of the falling mass. $F_{\text{net}} = Fg$ $= m_1g$ $= 1.5 \times 9.8$ $= 14.7 \text{ N in the positive direction}$
Calculate the mass being accelerated.	This net force has to accelerate not only the cart but also the falling mass. $m = m_1 + m_2$ $= 1.5 + 0.6$ $= 2.1 \text{ kg}$
Use Newton's second law to determine acceleration.	$a = \frac{F_{\text{net}}}{m}$ $= \frac{14.7}{2.1}$ $= 7.0 \text{ m s}^{-2} \text{ forwards}$

b If a frictional force of 4.2 N acts against the cart, what is the acceleration now?

Thinking	Working
Write down the data that is given. Apply the sign convention to vectors.	$m_1 = 1.5 \text{ kg}$ $m_2 = 0.6 \text{ kg}$ $g = 9.8 \text{ ms}^{-2}$ down $= +9.8 \text{ ms}^{-2}$ $F_{\text{fr}} = 4.2 \text{ N}$ backwards $= -4.2 \text{ N}$
Determine the forces acting on the system.	There are now two forces acting on the combined system of the cart and mass: the weight of the falling mass and friction. $F_{\text{net}} = F_{\text{g}} + F_{\text{fr}}$ $= 14.7 + (-4.2)$ $= 10.5 \text{ N}$ $= 10.5 \text{ N}$ in the positive direction
Use Newton's second law to determine acceleration.	$a = \frac{F_{\text{net}}}{m}$ $= \frac{10.5}{2.1}$ $= 5.0 \text{ ms}^{-2}$ forwards

Section 8.4 Review

KEY QUESTIONS SOLUTIONS

$$\begin{aligned}
 1 \quad a &= \frac{F_{\text{net}}}{m} \\
 &= \frac{158}{23.9} \\
 &= 6.61 \text{ ms}^{-2} \text{ north}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad m &= \frac{F_{\text{net}}}{a} \\
 &= \frac{352}{9.20} \\
 &= 38.3 \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad F_{\text{net}} &= \frac{(v - u)}{t} \\
 (v - u) &= \frac{F_{\text{net}} t}{m} \\
 v &= \frac{F_{\text{net}} t}{m} + u \\
 &= \frac{56.8 \times 3.50}{55.9} + 0 = \frac{56.8 \times 3.50}{55.9} \\
 &= 3.56 \text{ ms}^{-2} \text{ north}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad a &= \frac{F_{\text{net}}}{m} \\
 &= \frac{441}{45.0} \\
 &= 9.80 \text{ ms}^{-2} \text{ down}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad a &= \frac{F_{\text{net}}}{m} \\
 &= \frac{882}{90.0} \\
 &= 9.8 \text{ ms}^{-2} \text{ down}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{6} \quad F_{\text{net}} &= m \frac{(v - u)}{t} \\
 (v - u) &= \frac{F_{\text{net}} t}{m} \\
 v &= \frac{F_{\text{net}} t}{m} + u \\
 &= \frac{-45.5 \times 2.80}{60.0} + 2.67 \\
 &= 0.547 \text{ ms}^{-1} \text{ east}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7} \quad a &= \frac{F_{\text{net}}}{m} \\
 &= \frac{95.0}{0.0609} \\
 &= 1560 \text{ ms}^{-2} \text{ south}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{8} \quad m &= \frac{F_{\text{net}}}{a} \\
 &= \frac{565000}{7.20} \\
 &= 78500 \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{9} \quad F_{\text{net}} &= m \frac{(v - u)}{t} \\
 (v - u) &= \frac{F_{\text{net}} t}{m} \\
 v &= \frac{F_{\text{net}} t}{m} + u \\
 &= \frac{0.0823 \times 0.0105}{0.003} + 0 \\
 &= 0.288 \text{ ms}^{-1} \text{ north}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{10} \quad \mathbf{a} \quad \Sigma F_{\text{h}} &= 45 \text{ N south} + 25 \text{ N north} \\
 &= -45 + 25 \\
 &= -20 \\
 &= 20 \text{ N south}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad F_{\text{net}} &= ma \\
 20 &= 65a \\
 a &= 0.31 \text{ ms}^{-2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{11} \quad \mathbf{a} \quad F_{\text{net}} = F_{\text{g}} = mg &= 0.50 \times 9.8 = 4.9 \text{ N} \\
 a = \frac{F_{\text{net}}}{m} &= \frac{4.9}{(2.5 + 0.5)} = 1.6 \text{ ms}^{-2}
 \end{aligned}$$

$$\mathbf{b} \quad v = u + at = 0 + 1.6 \times 0.5 = 0.8 \text{ ms}^{-1}$$

$$\begin{aligned}
 \mathbf{c} \quad F_{\text{net}} = ma = F_{\text{g}} - F_{\text{f}} \\
 \text{so } F_{\text{net}} &= 4.9 - 4.3 = 0.6 \text{ N} \\
 \text{and } a &= \frac{F_{\text{net}}}{m} = \frac{0.6}{3} = 0.2 \text{ ms}^{-2}
 \end{aligned}$$

12 Determine the driving force provided by the truck based on the information for when it is empty.

$$\begin{aligned}
 F_{\text{net}} &= ma \\
 &= 2000 \times 2.0 \\
 &= 4000 \text{ N}
 \end{aligned}$$

Calculate the total mass of the truck for the new acceleration.

$$\begin{aligned}
 m &= \frac{F_{\text{net}}}{a} \\
 &= \frac{4000}{1.25} \\
 &= 3200 \text{ kg}
 \end{aligned}$$

So the mass of the boxes must be:

$$3200 - 2000 = 1200 \text{ kg}$$

$$\begin{aligned}
 \text{Number of boxes} &= \frac{1200}{300} \\
 &= 4 \text{ boxes}
 \end{aligned}$$

$$\begin{aligned}
 13 \quad F_{\text{net}} &= \text{thrust} - \text{weight of rocket} \\
 &= 1\,000\,000 - (50\,000 \times 9.8) \\
 &= 510\,000 \text{ N} \\
 a &= \frac{F_{\text{net}}}{m} \\
 &= \frac{510\,000}{50\,000} \\
 &= 10.2 \text{ ms}^{-2}
 \end{aligned}$$

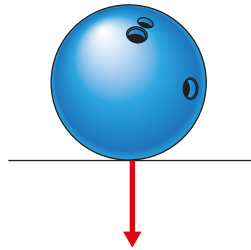
Section 8.5 Newton's third law

Worked example: Try yourself 8.5.1

APPLYING NEWTON'S THIRD LAW

In the diagram below, a bowling ball is resting on the floor and one of the forces is given. Copy the diagram into your book and complete the following:

- Label the given force using the system ' F on _____ by _____'.
- Label the reaction force to the given force using the system ' F on _____ by _____'.
- Draw the reaction force on the diagram, showing its size and location.



Thinking	Working
Identify the two objects involved in the action–reaction pair.	The bowling ball and the floor.
Identify which object is applying the force and which object is experiencing the force, for the force vector shown.	The force vector shown is a force from the bowling ball on the floor.
Use the system of labelling action and reaction forces ' F_{on} _____ by _____', to label the action force.	$F_{\text{on floor by bowling ball}}$
Use the system of labelling action and reaction forces ' F_{on} _____ by _____', to label the reaction force.	$F_{\text{on bowling ball by floor}}$
Use a ruler to measure the length of the action force and construct a vector arrow in the opposite direction with its tail on the point of application of the reaction force.	

Section 8.5 Review

KEY QUESTIONS SOLUTIONS

- There is a force on the hammer by the nail, and a force on the nail by the hammer. These two forces are equal in magnitude and opposite in direction.
- The force arrow shown is the $F_{\text{on the astronaut by the Earth}}$
 - The reaction force must act on the other object, so in this case it is the $F_{\text{on the Earth by the astronaut}}$
- The force on the hand by the water. The swimmer creates the action force by pushing on the water, but the reaction force acts on the swimmer which moves him in the direction of his motion.
- The force on the balloon by the escaping air. The balloon's elasticity compresses the air inside and pushes it out of the mouth of the balloon. This is the action force. The air must therefore exert an equal and opposite forwards force on the balloon, which in turn moves the balloon around the room.
- The reaction force is on the racquet by the ball, resulting in a force of 100 N east.
- The boat exerts an equal and opposite reaction force, i.e. 140 N in the opposite direction to the leaping fisherman.
 - $a = \frac{F_{\text{net}}}{m} = \frac{140}{40} = 3.5 \text{ ms}^{-2}$ in the opposite direction to the fisherman
 - Acceleration of the fisherman: $a = \frac{F_{\text{net}}}{m} = \frac{140}{70} = 2.0 \text{ ms}^{-2}$
 Speed of the fisherman: $v = 0 + 2 \times 0.5 = 1.0 \text{ ms}^{-1}$
 Speed of the boat: $v = 0 + 3.5 \times 0.5 = 1.8 \text{ ms}^{-1}$
- The astronaut should throw the tool kit in the opposite direction from the ship. By throwing the tool kit, there is an action force on the tool kit by the astronaut which is directed away from the ship. According to Newton's third law, there will be a reaction force on the astronaut by the tool kit that will be in the opposite direction, i.e. towards the ship.
- Tania is correct. For an action–reaction pair, the action force is a force on object A by object B, and the reaction force is a force on object B by object A. That is, the two forces act on different objects. In this case, both the weight force and the normal force are acting on the same object: the lunch box.

Section 8.6 Impulse and force

Worked example: Try yourself 8.6.1

CALCULATING THE FORCE AND IMPULSE

A student drops a 56.0 g egg onto a table from a height of 60 cm. Just before the egg hits the table, the velocity of the egg is 3.43 ms^{-1} down. The egg's final velocity is zero as it smashes on the table. The time it takes for the egg to change its velocity to zero is 3.55 ms.

a Calculate the change in momentum of the egg.	
Thinking	Working
Ensure that the variables are in their standard units.	$m = 0.0560 \text{ kg}$ $u = 3.43 \text{ ms}^{-1}$ down $v = 0 \text{ ms}^{-1}$
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 0.0560 \text{ kg}$ $u = -3.43 \text{ ms}^{-1}$ $v = 0 \text{ ms}^{-1}$
Apply the equation for change in momentum.	$\Delta p = m(v - u)$ $= 0.0560 \times (0 - (-3.43))$ $= 0.192 \text{ kg ms}^{-1}$
Refer to the sign and direction convention to determine the direction of the change in momentum.	$\Delta p = 0.192 \text{ kg ms}^{-1}$ up

b Calculate the impulse of the egg.	
Thinking	Working
Using the answer to part (a), apply the equation for impulse.	$I = \Delta p$ $= 0.192 \text{ kg ms}^{-1}$
Refer to the sign and direction convention to determine the direction of the impulse.	$I = 0.192 \text{ kg ms}^{-1}$ up
c Calculate the average force that acts to cause the impulse.	
Thinking	Working
Use the answer to part (b). Ensure that the variables are in their standard units.	$I = 0.192 \text{ kg ms}^{-1}$ $\Delta t = 3.55 \times 10^{-3} \text{ s}$
Apply the equation for force.	$F\Delta t = I$ $F = \frac{I}{\Delta t}$ $= \frac{0.192}{3.55 \times 10^{-3}}$ $= 54.1 \text{ N}$
Refer to the sign and direction convention to determine the direction of the force.	$F = 54.1 \text{ N}$ up

Worked example: Try yourself 8.6.2

CALCULATING THE FORCE AND IMPULSE (SOFT LANDING)

A student drops a 56.0 g egg into a mound of flour from a height of 60 cm. Just before the egg hits the mound of flour, the velocity of the egg is 3.43 m s^{-1} down. The egg's final velocity is zero as it sinks into the mound of flour. The time it takes for the egg to change its velocity to zero is 0.325 s.

a Calculate the change in momentum of the egg.	
Thinking	Working
Ensure that the variables are in their standard units.	$m = 0.0560 \text{ kg}$ $u = 3.43 \text{ m s}^{-1}$ down $v = 0 \text{ m s}^{-1}$
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 0.0560 \text{ kg}$ $u = -3.43 \text{ m s}^{-1}$ $v = 0 \text{ m s}^{-1}$
Apply the equation for change in momentum.	$\Delta p = m(v - u)$ $= 0.0560 \times (0 - (-3.43))$ $= 0.192 \text{ kg ms}^{-1}$
Refer to the sign and direction convention to determine the direction of the change in momentum.	$\Delta p = 0.192 \text{ kg ms}^{-1}$ up
b Calculate the impulse of the egg.	
Thinking	Working
Using the answer to part (a), apply the equation for impulse.	$I = \Delta p$ $= 0.192 \text{ kg ms}^{-1}$
Refer to the sign and direction convention to determine the direction of the impulse.	$I = 0.192 \text{ kg ms}^{-1}$ up

c Calculate the average force that acts to cause the impulse.

Thinking

Using the answer to part (b), ensure that the variables are in their standard units.

Apply the equation for force.

Refer to the sign and direction convention to determine the direction of the force.

Working

$$I = 0.192 \text{ kg ms}^{-1}$$

$$\Delta t = 0.325 \text{ s}$$

$$F\Delta t = I$$

$$F = \frac{I}{\Delta t}$$

$$= \frac{0.192}{0.325}$$

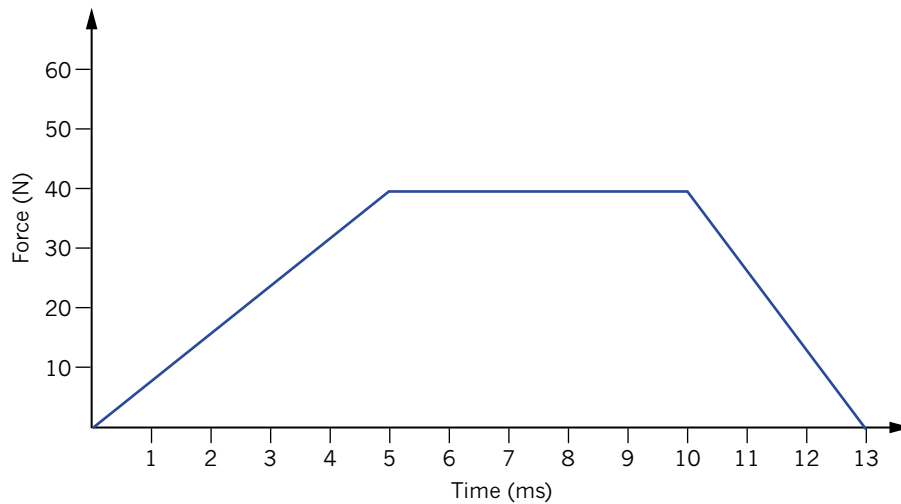
$$= 0.591 \text{ N}$$

$$F = 0.591 \text{ N up}$$

Worked example: Try yourself 8.6.3

CALCULATING THE TOTAL IMPULSE FROM A CHANGING FORCE

A student records the force acting on a tennis ball as it bounces off a hard concrete floor over a period of time. The graph shows the forces acting on a ball during its collision with the concrete floor.

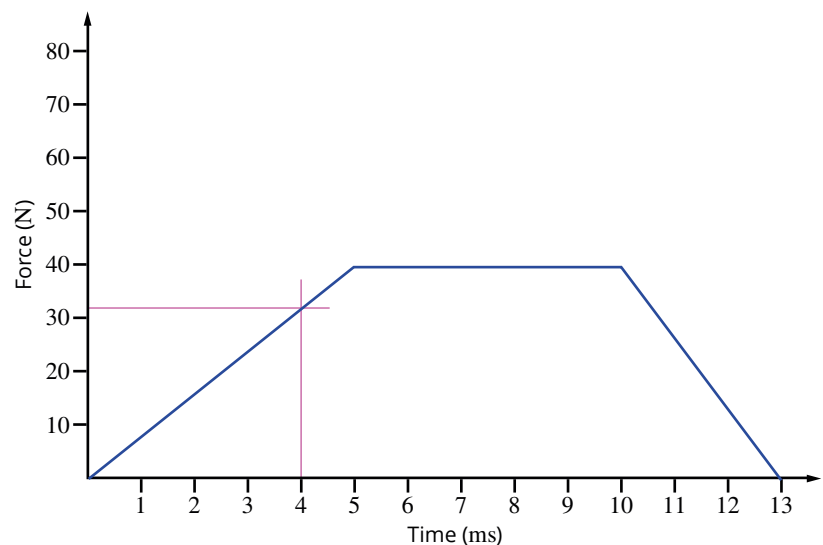


a Determine the force acting on the ball at a time of 4.0 ms.

Thinking

From the 4.0 ms point on the x-axis go up to the line of the graph, then across to the y-axis.

Working



The force is estimated by reading the intercept of the y-axis

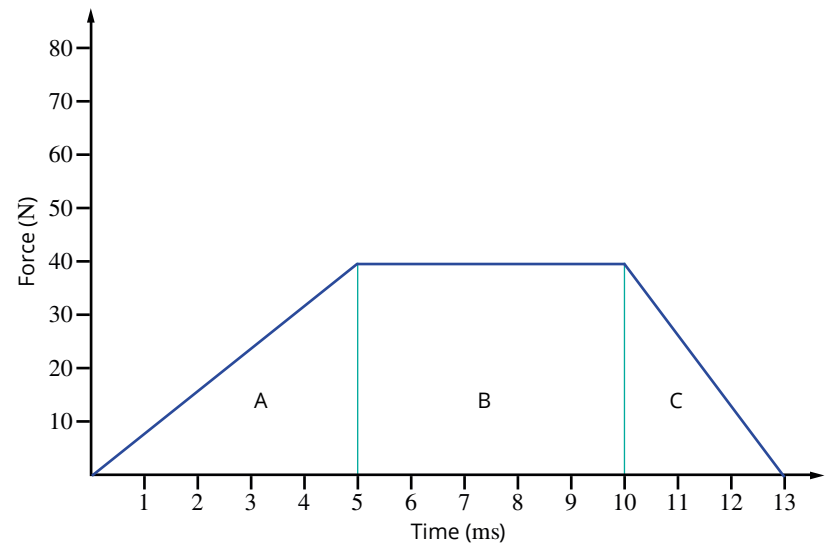
$$F = 32 \text{ N}$$

b Calculate the total impulse of the ball over the 13 ms period of time.

Thinking

Break the area under the graph into sections for which you can calculate the area.

In this case, the graph can be broken into three sections: A, B and C.



Calculate the area of the three sections A, B and C using the equations for the area of a triangle and the area of a rectangle.

$$\begin{aligned}
 \text{area} &= A + B + C \\
 &= \left(\frac{1}{2}b \times h\right) + (b \times h) + \left(\frac{1}{2}b \times h\right) \\
 &= \left[\frac{1}{2} \times (5 \times 10^{-3}) \times 40\right] + [(5 \times 10^{-3}) \times 40] + \left[\frac{1}{2} \times (3 \times 10^{-3}) \times 40\right] \\
 &= 0.1 + 0.2 + 0.06 \\
 &= 0.36
 \end{aligned}$$

The total impulse is equal to the area.

$$\begin{aligned}
 I &= \text{area} \\
 &= 0.36 \text{ kg m s}^{-1}
 \end{aligned}$$

Apply the sign and direction convention for motion in one dimension vertically.

$$I = 0.36 \text{ kg m s}^{-1} \text{ up}$$

Section 8.6 Review

KEY QUESTIONS SOLUTIONS

1 a $\Delta p = m(v - u)$
 $= 45.0 \times (12.5 - 2.45)$
 $= 452 \text{ kg m s}^{-1} \text{ east}$

b $I = \Delta p$
 $= 452 \text{ kg m s}^{-1} \text{ east}$

c $F_{\text{av}} \Delta t = I$
 $F_{\text{av}} = \frac{I}{\Delta t}$
 $= \frac{452}{3.50}$
 $= 129 \text{ N east}$

2 Airbags are designed to increase the duration of the collision, which changes the momentum of a person's head during a car accident. Increasing the duration of the collision decreases the force, which reduces the severity of injury.

- 3 a** $\Delta p = m(v - u)$
 $= 0.075 \times (0 - (-15.6))$
 $= 1.17 \text{ kg m s}^{-1} \text{ east}$
- b** $I = \Delta p$
 $= 1.17 \text{ kg m s}^{-1} \text{ east}$
- c** $F_{\text{av}} \Delta t = I$
 $F_{\text{av}} = \frac{I}{\Delta t}$
 $= \frac{1.17}{0.100}$
 $= 11.7 \text{ N east}$
- 4** $F_{\text{av}} \Delta t = I$
 $F_{\text{av}} = \frac{I}{\Delta t}$
 $= \frac{1.17}{0.300}$
 $= 3.90 \text{ N east}$
- 5 a** Impulse $= F \Delta t = \Delta p$
 $= 0.200 \times 45$
 $= 9.0 \text{ kg m s}^{-1}$
- b** $F = \frac{9.0 \text{ kg m s}^{-1}}{0.05 \text{ s}}$
 $= 180 \text{ N in the direction of the ball's travel}$
- c** 180 N in the opposite direction to the ball's travel.
- 6 a** Maximum force = 1200 N
- b** Impulse $= F \Delta t = \text{area under force-time graph} = 63 \text{ N s}$
- 7 a** $\Delta p = m(v - u)$
 $= (0.025) \times (0 - 50)$
 $= 1.25 \text{ kg m s}^{-1} \text{ opposite in direction to its initial velocity}$
- b** Impulse $= F \Delta t = \Delta p$
 $= 1.25 \text{ kg m s}^{-1} \text{ opposite in direction to its initial velocity}$
- c** $v^2 = u^2 + 2as$
 $0 = 50^2 + 2a(2.0 \times 10^{-2})$
 $a = -6.25 \times 10^4 \text{ m s}^{-2}$
 $F = ma$
 $= 0.025(-6.25 \times 10^4)$
 $= 1.6 \times 10^3 \text{ N in the opposite direction to the initial velocity of the arrow}$
- 8 a** The crash helmet is designed so that the stopping time is increased by the collapsing shell during impact. This will reduce the force, as impulse $= F \Delta t = \Delta p$.
- b** No. A rigid shell would reduce the stopping time, therefore increasing the force.

Section 8.7 Mass and weight

Section 8.7 Review

KEY QUESTIONS SOLUTIONS

- 50 kg. The mass of an object does not depend on its environment.
- 60 kg is the student's mass, not her weight. Weight is a force and so it is measured in newtons. On Earth, her weight will be 9.8 times larger than her mass.
- $F_g = mg = 75 \times 9.8 = 735 \text{ N}$
- $$m = \frac{F_g}{g}$$

$$= \frac{34.3}{9.8}$$

$$= 3.5 \text{ kg}$$
- $$F_g = mg$$

$$= 3.5 \times 1.6$$

$$= 5.6 \text{ N}$$
- The mass of the hammer remains constant at 1.5 kg.
The weight of the hammer on Mars is $F_g = mg = 1.5 \times 3.6 = 5.4 \text{ N}$.
- The weight of any object will be less on the Moon compared with its weight on the Earth as gravity is weaker on the Moon, due to its smaller mass.

CHAPTER 8 REVIEW

- No, a force has not pushed the passengers backwards. Since the passengers have inertia, as the train has started moving forwards the passengers' masses resist the change in motion. According to Newton's first law, their bodies are simply maintaining their original state of being motionless until an unbalanced force acts to accelerate them.
- D. An object travelling at a constant velocity will do so without any force acting.
- $$m = \frac{F_{\text{net}}}{a}$$

$$= \frac{352}{9.20}$$

$$= 38.3 \text{ kg}$$
- $$a = \frac{F_{\text{net}}}{m}$$

$$= \frac{3550}{657}$$

$$= 5.40 \text{ ms}^{-2} \text{ north}$$
- $$a = \frac{F_{\text{net}}}{m}$$

$$= \frac{150}{100}$$

$$= 1.5 \text{ ms}^{-2}$$
- $$F_{\text{net}} = 150 - 45$$

$$= 105 \text{ N}$$

$$a = \frac{F_{\text{net}}}{m}$$

$$= \frac{105}{100}$$

$$= 1.05 \text{ ms}^{-2}$$

- 7 Using Newton's second law:

$$\begin{aligned} F_{\text{net}} &= ma \\ &= 100 \times 0.6 \\ &= 60.0\text{N} \end{aligned}$$

Net force is also given by the sum of individual forces:

$$\begin{aligned} F_{\text{net}} &= 150 - F_{\text{fr}} \\ F_{\text{fr}} &= 150 - F_{\text{net}} \\ &= 150 - 60 \\ &= 90\text{N} \end{aligned}$$

- 8 Using Newton's second law:

$$\begin{aligned} F_{\text{net}} &= ma \\ &= 125 \times 0.800 \\ &= 100\text{N} \end{aligned}$$

Net force is also given by the sum of individual forces:

$$\begin{aligned} F_{\text{net}} &= F_{\text{forwards}} - 30 \\ F_{\text{forwards}} &= F_{\text{net}} + 30 \\ &= 100 + 30 \\ &= 130\text{N} \end{aligned}$$

- 9 $F = m \frac{(v - u)}{\Delta t}$

$$\begin{aligned} (v - u) &= \frac{F\Delta t}{m} \\ v &= \frac{F\Delta t}{m} + u \\ &= \frac{-62.0 \times 2.00}{4.0} + 3.75 \\ &= -5.11 \text{ ms}^{-1} \end{aligned}$$

$$\therefore v = 5.11 \text{ ms}^{-1} \text{ west}$$

- 10 Newton's third law states that every action has an equal and opposite reaction. Therefore, the reaction of the board acting on the student results in a force of 75.0N north.

11 $\Delta p = m(v - u)$

$$\begin{aligned} &= 155 \times (3.25 - 6.50) \\ &= 504 \text{ kg m s}^{-1} \text{ west} \end{aligned}$$

12 $\Delta p = mv - mu$

$$\begin{aligned} &= (25.5 \times -2.25) - (25.5 \times 6.40) \\ &= -221 \text{ kg m s}^{-1} \\ &= 221 \text{ kg m s}^{-1} \text{ backwards} \end{aligned}$$

13 $\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$

$$\begin{aligned} m_1 u_1 + m_2 u_2 &= m_1 v_1 + m_2 v_2 \\ (40.0 \times 0) + (154 \times 0) &= (40.0 \times 2.15) + 154 v_2 \\ 154 v_2 &= 0 + -86 \\ v_2 &= \frac{-86}{154} \\ &= -0.558 \text{ ms}^{-1} \end{aligned}$$

The astronaut moves backwards at 0.558 ms^{-1} .

14 $\Delta v = \text{final velocity} - \text{initial velocity}$

$$= v - u$$

$$= v + (-u)$$

$$= 5.00 \text{ ms}^{-1} \text{ north} + 4.00 \text{ ms}^{-1} \text{ east}$$

The magnitude of the change in velocity is calculated using Pythagoras' theorem:

$$\Delta v^2 = 5.00^2 + 4.00^2$$

$$= 25.0 + 16.0$$

$$\Delta v = \sqrt{41.0}$$

$$= 6.40 \text{ ms}^{-1}$$

Use trigonometry to calculate the angle of the change in momentum.

$$\tan \theta = \frac{4.00}{5.00}$$

$$\theta = \tan^{-1}(0.9)$$

$$= 38.7^\circ$$

$$\Delta v = 6.40 \text{ ms}^{-1} \text{ N } 38.7^\circ \text{ E}$$

The magnitude of the change in momentum is calculated using:

$$\Delta p = mv - mu$$

$$= m(v - u)$$

$$= m\Delta v$$

$$= 75.0 \times 6.40$$

$$= 480 \text{ kg ms}^{-1}$$

$$= 480 \text{ kg ms}^{-1} \text{ N } 38.7^\circ \text{ E}$$

15 $\Delta p = m(v - u)$

$$= 0.300 \times (0 - (-5.60))$$

$$= 1.68 \text{ kg ms}^{-1} \text{ east}$$

$$F_{\text{av}} \Delta t = \Delta p$$

$$F_{\text{av}} = \frac{\Delta p}{\Delta t}$$

$$= \frac{1.68}{1.00}$$

$$= 1.68 \text{ N east}$$

16 When a car travelling at a very fast speed comes to a complete halt, the occupants will experience a force applied to them of $F_{\text{net}} = ma = \frac{l}{\Delta t}$. In this case, the mass of the occupants and the impulse (Δp) is always the same. The amount

of force acting upon the occupants therefore depends entirely upon the value of their acceleration, or the time over which a collision takes place; a short collision will involve a large deceleration and therefore a large force applied to the occupants. By designing the bonnet of the car to be long and to crumple, a collision will deform this metal and slow the car down before the impact reaches the occupants, thereby reducing the total force applied to the occupants.

A frame made out of metal that is of medium rigidity is best for this purpose. If the metal of the car frame crumpled too easily then it would not slow down the collision very much, and the force applied to occupants would be high. If the car frame remained too rigid in the event of a collision, then none of the energy of the crash would go into deforming the metal, and the force would be passed entirely to the car occupants. The ideal metal shell should be as strong as possible, so that it would still crumple in the event of a collision. Car manufacturers subject their cars to crash testing during their design in order to optimise this collision time.

17 a $p = mv$

$$pi = 70.0 \times 5.0$$

$$= 350 \text{ kg ms}^{-1}$$

b $l = \Delta p$

$$= (0 - 350)$$

$$= -350 \text{ kg ms}^{-1}$$

$$F_{\text{net}} = \frac{l}{\Delta t}$$

$$= \frac{-350}{0.350}$$

$$= -1000 \text{ N}$$

Hence 1000 N of force would act on Young.

$$\begin{aligned}
 \text{c } F_{\text{net}} &= \frac{I}{\Delta t} \\
 &= \frac{-350}{7.00 \times 10^{-3}} \\
 &= -50000 \text{ N}
 \end{aligned}$$

Hence 50000 N of force would act on Young's head. Clearly a helmet is important in this case.

d As well as reducing the force acting on the user's head, a crash helmet spreads out the area of the head over which the force is applied, which reduces the risk of penetration of the skull.

18 $\Delta p = \text{impulse} = \text{area under } F-t \text{ graph} = 0.5 \times 0.04 \times 500 = 10 \text{ kg ms}^{-1}$

19 As the bat and ball form an isolated system, momentum is conserved. The gain in momentum of the ball is equal to the loss of momentum of the bat. Hence:

$$\Delta p = 10 \text{ kg ms}^{-1}$$

20 $\Delta p = m\Delta v$

Therefore:

$$\begin{aligned}
 \Delta v &= \frac{\Delta p}{m} \\
 &= \frac{10}{0.170} \\
 &= 58.8 \\
 &= 59 \text{ ms}^{-1}
 \end{aligned}$$

21 $F_g = mg = 10 \times 9.8 = 98 \text{ N}$

22 $m = \frac{F_g}{g}$
 $= \frac{20.6}{9.8}$
 $= 2.1 \text{ kg}$

23 a mass = 85 kg

b mass = 85 kg

c $F_g = mg = 85 \times 3.6 = 306 \text{ N down}$

24 The object's greatest weight is when it is on Earth. The second greatest weight is on Mars, and its least weight is when it is on the Moon.

Chapter 9 Work, energy and power

Section 9.1

Worked example: Try yourself 9.1.1

CALCULATING WORK

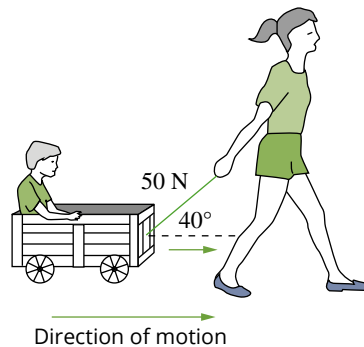
A person pushes a heavy wardrobe from one room to another by applying a force of 50 N for a distance of 5 m. Calculate the amount of work done.

Thinking	Working
Recall the definition of work.	$W = Fs$
Substitute in the values for this situation.	$W = 50 \times 5$
Solve the problem, giving an answer with appropriate units.	$W = 250 \text{ J}$

Worked example: Try yourself 9.1.2

WORK WITH FORCE AND DISPLACEMENT AT AN ANGLE

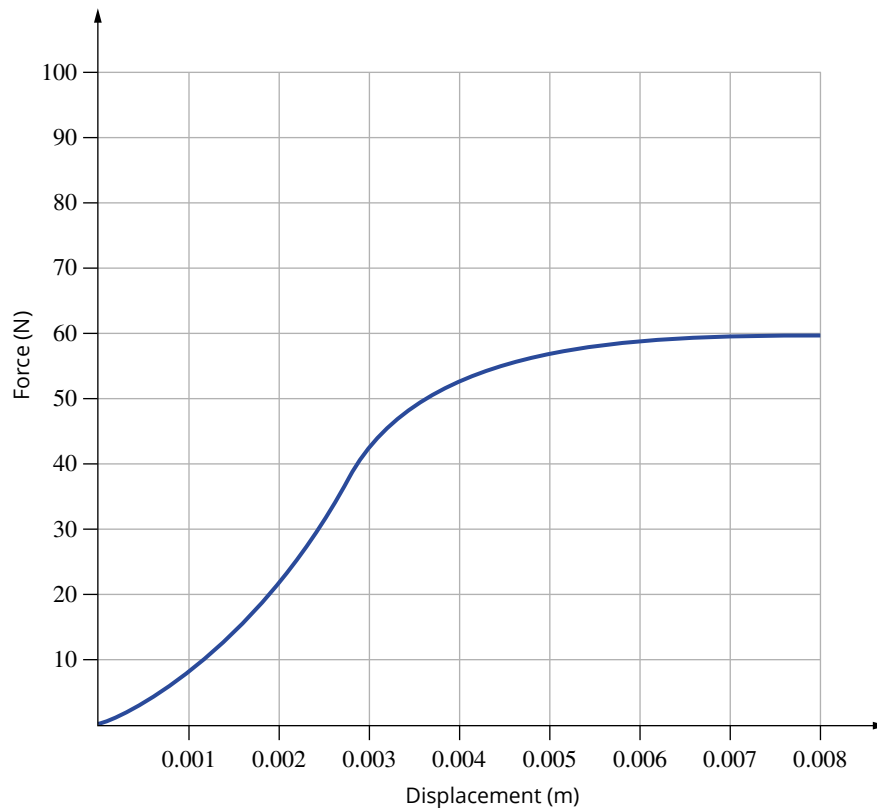
A girl pulls her brother along in a trolley for a distance of 30 m, as shown. Calculate the work done on the box. Give your answer correct to three significant figures.



Thinking	Working
Determine values for F , s and θ .	$F = 50 \text{ N}$ $s = 30 \text{ m}$ $\theta = 40^\circ$
Recall the work equation.	$W = Fs \cos \theta$
Substitute values into the work equation.	$W = 50 \times 30 \times \cos 40^\circ$
State the answer with the correct units.	$W = 1150 \text{ J}$

Worked example: Try yourself 9.1.3
WORK FROM THE AREA UNDER A FORCE–DISPLACEMENT GRAPH

While jogging, a person's shoes compress by an average of 3 mm with each step. Use the force–displacement graph for a sports shoe to estimate how much work is done on the shoe with each step. Give your answer to the nearest 0.01 J.



Thinking	Working
Calculate the work value of each grid square.	The dimensions of a grid square are: Force: 10 N, displacement: 0.001 m Area of 1 square = $10 \times 0.001 = 0.01$ J
Count the number of grid squares under the curve up to a distance of 3 mm or 0.003 m. Only count grid squares that are more than half under the curve. If the curve cuts a square in half, count every second one.	Number of squares = 5
Multiply the number of grid squares under the curve by the work value of each grid square.	$W = 5 \times 0.01 = 0.05$ J

Section 9.1 Review

- $W = Fs = 500 \times 20 = 10\,000$ J
- The person exerts a force on the wall but the wall has no displacement ($s = 0$), so no work is done.
- $A = \frac{1}{2} \times b \times h = \frac{1}{2} \times 0.015 \times 200 = 1.5$ J
- $W = Fs$
 $2700 = F \times 150$
 $F = 18$ N

- 5 $W = Fs \cos \theta$
 $= 80 \times 5.0 \times \cos 40^\circ$
 $= 310\text{J}$
- 6 The equation $W = Fs$ applies to situations where the applied force is constant. Since a spring obeys Hooke's law, the force required to compress a spring is not constant.
- 7 Since the box does not move, no work is done.
- 8 The area under the graph is a triangle.
 $A = \frac{1}{2} \times b \times h = \frac{1}{2} \times 6 \times 10^{-3} \times 1200 = 3.6\text{J}$
- 9 Work done is the area under the force-displacement graph. This can be calculated by approximating the area to a geometrical shape or by counting squares.
 1 square = $20\text{N} \times 10\text{mm} = 20\text{N} \times 0.01\text{m} = 0.2\text{J}$
 W_A is approximately 15 squares $\times 0.2 = 3.0\text{J}$
 W_B is approximately 12 squares $\times 0.2 = 2.4\text{J}$
 W_C is approximately 5 squares $\times 0.2 = 1.0\text{J}$
- 10 a $A = \frac{1}{2}bh = \frac{1}{2} \times 0.04 \times 400 = 8\text{J}$
 b $A = \frac{1}{2}bh = \frac{1}{2} \times 0.03 \times 300 = 4.5\text{J}$
 c As the basketball bounces, some energy is lost as heat and sound so the work when the ball rebounds is less than the work done when the ball compresses.

Section 9.2 Kinetic energy

Worked example: Try yourself 9.2.1

CALCULATING KINETIC ENERGY

A person crossing the street is walking at 5.0 km h^{-1} . If the person has a mass of 80 kg , calculate their kinetic energy. Give your answers correct to two significant figures.	
Thinking	Working
Convert the person's speed to ms^{-1} .	$5\text{ km h}^{-1} = \frac{5\text{ km}}{1\text{ h}} = \frac{5000\text{ m}}{3600\text{ s}} = 1.4\text{ ms}^{-1}$
Recall the equation for kinetic energy.	$E_k = \frac{1}{2}mv^2$
Substitute the values for this situation into the equation.	$E_k = \frac{1}{2} \times 80 \times 1.4^2$
State the answer with appropriate units.	$E_k = 78\text{J}$

Worked example: Try yourself 9.2.2
CALCULATING KINETIC ENERGY CHANGES

As a bus with a mass of 10 tonnes approaches a school it slows from 60 km h^{-1} to 40 km h^{-1} .

a Calculate the work done by the brakes in the bus. Give your answers correct to two significant figures.	
Thinking	Working
Convert the values into SI units.	$u = 60 \text{ km h}^{-1}$ $= \frac{60 \text{ km}}{1 \text{ h}}$ $= \frac{60000 \text{ m}}{3600 \text{ s}}$ $= 17 \text{ m s}^{-1}$ $v = 40 \text{ km h}^{-1}$ $= \frac{40 \text{ km}}{1 \text{ h}}$ $= \frac{40000 \text{ m}}{3600 \text{ s}}$ $= 11 \text{ m s}^{-1}$ $m = 10 \text{ tonnes}$ $= 10\,000 \text{ kg}$
Recall the work–energy theorem.	$W = \frac{1}{2} \times mv^2 - \frac{1}{2} \times mu^2$
Substitute the values for this situation into the equation.	$W = \frac{1}{2}(10\,000 \times 11^2) - \frac{1}{2}(10\,000 \times 17^2)$
State the answer with appropriate units.	$W = -840\,000 \text{ J} = -840 \text{ kJ}$ <p>Note: the negative value indicates that the work has caused the kinetic energy to decrease.</p>
b The bus travels 40 m as it decelerates. Calculate the average force applied by the truck's brakes.	
Thinking	Working
Recall the definition of work.	$W = Fs$
Substitute the values for this situation into the equation. Note: The negative has been ignored as work is a scalar.	$840\,000 \text{ J} = F \times 40 \text{ m}$
Transpose the equation to find the answer.	$F = \frac{W}{s} = \frac{840\,000 \text{ J}}{40 \text{ m}} = 21\,000 \text{ N}$

Worked example: Try yourself 9.2.3
CALCULATING SPEED FROM KINETIC ENERGY

A 300 kg motorbike has 150 kJ of kinetic energy. Calculate the speed of the motorbike in km h^{-1} . Give your answer correct to two significant figures.	
Thinking	Working
Recall the equation for kinetic energy.	$E_k = \frac{1}{2} mv^2$
Transpose the equation to make v the subject.	$v = \sqrt{\frac{2E_k}{m}}$
Substitute the values for this situation into the equation.	$v = \sqrt{\frac{2 \times 150\,000}{300}} = 31.6 \text{ m s}^{-1}$
State the answer with appropriate units.	$v = 31.6 \times 3.6 = 114 \text{ km h}^{-1}$

Section 9.2 Review

KEY QUESTIONS SOLUTIONS

1 $80 \text{ km h}^{-1} = 22.22 \text{ ms}^{-1}$

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}230 \times 22.22^2 = 56\,779 \text{ J or } 57 \text{ kJ}$$

2 $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2} \times 1500 \times 28^2 - \frac{1}{2} \times 1500 \times 17^2 = 370\,000 \text{ J}$

3 $v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 5000}{72 + 9}} = 11 \text{ ms}^{-1} = 40 \text{ km h}^{-1}$

4 $E_k = \frac{1}{2}mv^2 \therefore E_{k_{\text{new}}} \propto m$

So doubling the mass causes E_k to increase by a factor of 2 as well.

5

If $E_{k1} = \frac{1}{2}mv_1^2$ then

$$E_{k2} = \frac{1}{2}m(v_2)^2$$

$$= \frac{1}{2}m(3v_1)^2$$

$$= 9 \times \frac{1}{2}m(v_1)^2$$

$$= 9 \times E_{k1}$$

Hence the kinetic energy of the object is increased by a factor of 9.

6 The work–energy theorem defines work as the change in kinetic energy, so Lauren needs to increase her kinetic energy by 1000 J.

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = 1000$$

$$\frac{1}{2} \times 57.0 \times (v^2 - 0.500^2) = 1000$$

$$v^2 - 0.500^2 = 35.09$$

$$v^2 = 35.34$$

$$\text{So } v = 5.94 \text{ ms}^{-1}$$

7 $E_k = \frac{1}{2}mv^2$

Convert km h^{-1} to ms^{-1} :

For car 1 (travelling at 65 km h^{-1})

$$65.0 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{\text{km}} \times \frac{\text{h}}{3600 \text{ s}}$$

$$= \frac{65.0}{3.60}$$

$$= 18.06 \text{ ms}^{-1}$$

For car 2 (travelling at 60 km h^{-1})

$$\frac{60.0}{3.60} = 16.67 \text{ ms}^{-1}$$

car 1:

$$E_{k1} = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 1800 \times 18.06^2$$

$$= 2.94 \times 10^5 \text{ J}$$

car 2:

$$\begin{aligned}
 E_{k1} &= \frac{1}{2}mv^2 \\
 &= \frac{1}{2} \times 1800 \times 16.67^2 \\
 &= 2.50 \times 10^5 \text{ J}
 \end{aligned}$$

i.e. car 1 has an extra 44 000 J of energy just by going 5 km h⁻¹ faster. Energy differences such as these become very significant in the event of a collision.

Section 9.3 Elastic and inelastic collisions

Worked example: Try yourself 9.3.1

ELASTIC OR INELASTIC COLLISION?

A 200g snooker ball with initial velocity 9.0 ms ⁻¹ to the right collides with a stationary snooker ball of mass 100g. After the collision, both balls are moving to the right and the 200g ball has a speed of 3.0 ms ⁻¹ . Show calculations to test whether or not the collision is inelastic.	
Thinking Use conservation of momentum to find the final velocity of the 100g ball.	Working Taking to the right as positive: $p_{i100} + p_{i200} = p_{f100} + p_{f200}$ $mv_{i100} + mv_{i200} = mv_{f100} + mv_{f200}$ $(0.1 \times 0) + (0.2 \times 9.0) = (0.1)v_{f100} + (0.2 \times 3.0)$ $v_{f100} = \frac{0 + 1.8 - 0.6}{0.1}$ $v_{f100} = 12 \text{ ms}^{-1}$
Calculate the initial kinetic energy before the collision.	$E_{ki200} = \frac{1}{2}mv_i^2$ $= \frac{1}{2} \times (0.2) \times (9.0)^2$ $= 8.1 \text{ J}$ $E_{ki100} = \frac{1}{2}mv_i^2$ $= \frac{1}{2} \times (0.1) \times (0)^2$ $= 0 \text{ J}$ $E_{ki} = E_{ki100} + E_{ki200}$ $= (0) + (8.1)$ $= 8.1 \text{ J}$
Calculate the final kinetic energy of the balls after the collision.	$E_{kf} = E_{kf100} + E_{kf200}$ $= \frac{1}{2}mv_f^2 + \frac{1}{2}mv_f^2$ $= \frac{1}{2} \times (0.1) \times (12)^2 + \frac{1}{2} \times (0.2) \times (3.0)^2$ $= 7.2 + 0.9$ $= 8.1 \text{ J}$
Compare the kinetic energy before and after the collision to determine whether or not the collision is inelastic.	The kinetic energy after the collision is the same as the kinetic energy before the collision. The collision is perfectly elastic.

Section 9.3 Review

KEY QUESTIONS SOLUTIONS

1 An elastic collision is one in which the kinetic energy of the objects involved before the collision is exactly equal to the kinetic energy of the objects after the collision. An inelastic collision is one where the total kinetic energy of the objects after the collision is lower. The law of conservation of energy places no restrictions on which type of energy is present; even though kinetic energy is lower, overall energy is still conserved.

2 a Take east as positive

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

$$1500 \times 12.0 + 2500 \times (-16.0) = (1500 + 2500)v_3$$

$$-22000 = 4000v_3$$

$$v_3 = -5.50 \text{ m s}^{-1}$$

So 5.50 m s^{-1} west

b Initial kinetic energy = $\frac{1}{2} \times 1500 \times 12.0^2 + \frac{1}{2} \times 2500 \times 16.0^2$

$$= 428000 \text{ J}$$

Final kinetic energy = $\frac{1}{2} \times 4000 \times 5.50^2$

$$= 60500 \text{ J}$$

Since the final kinetic energy is less than the initial kinetic energy, the collision is inelastic.

3 $mv_1 = (m + m + m + m + m)v_2$

$$0.0400 \times 1.50 = 5 \times 0.0400 \times v_2$$

$$v_2 = 0.300 \text{ m s}^{-1}$$

Initial kinetic energy = $\frac{1}{2} \times 0.0400 \times 1.50^2$

$$= 0.0450 \text{ J}$$

Final kinetic energy = $\frac{1}{2} \times 0.2 \times 0.300^2$

$$= 0.00900 \text{ J}$$

So the collision is inelastic

4 $p_i(\text{truck}) + p_i(\text{car}) = p_f(\text{truck}) + p_f(\text{car})$

$$0.20 \times 0.30 + 0.10 \times 0.20 = 0.20v_f + 0.10 \times 0.30$$

$$0.08 = 0.2v_f + 0.03$$

$$0.2v_f = 0.05$$

$$v_f = 0.05 \div 0.2$$

$$= 0.25 \text{ m s}^{-1}$$

5 $E_{ki} = \frac{1}{2}m_t(v_{ti})^2 + \frac{1}{2}m_c(v_{ci})^2$

$$= \frac{1}{2} \times 0.200 \times 0.300^2 + \frac{1}{2} \times 0.100 \times 0.200^2$$

$$= 0.009 + 0.002$$

$$= 1.10 \times 10^{-2} \text{ J}$$

6 $E_{kf} = \frac{1}{2}m_t(v_{tf})^2 + \frac{1}{2}m_c(v_{cf})^2$

$$= \frac{1}{2} \times 0.200 \times 0.250^2 + \frac{1}{2} \times 0.100 \times 0.300^2 \text{ J}$$

$$= 6.25 \times 10^{-3} + 4.50 \times 10^{-3}$$

$$= 1.08 \times 10^{-2} \text{ J}$$

7 a The total kinetic energy before the collision is **more than** the total kinetic energy after the collision.

b The kinetic energy of the system of toys **is not** conserved.

c The total energy of the system of toys **is** conserved.

d The total momentum of the system of toys **is** conserved.

e The collision **is not** perfectly elastic because **kinetic energy** is not conserved.

Section 9.4 Gravitational potential energy

Worked example: Try yourself 9.4.1

CALCULATING GRAVITATIONAL POTENTIAL ENERGY

A person doing their grocery shopping lifts a 5 kg grocery bag to a height of 30 cm. Calculate the gravitational potential energy of the grocery bag at this height. Give your answer correct to two significant figures.	
Thinking	Working
Recall the formula for gravitational potential energy.	$E_g = mg\Delta h$
Substitute the values for this situation into the equation.	$E_g = 5 \times 9.80 \times 0.3$
State the answer with appropriate units and significant figures.	$E_g = 15\text{ J}$

Worked example: Try yourself 9.4.2

CALCULATING GRAVITATIONAL POTENTIAL ENERGY RELATIVE TO A REFERENCE LEVEL

A father picks up his baby from its bed. The baby has a mass of 6.0 kg and the mattress of the bed is 70 cm above the ground. When the father holds the baby in his arms, it is 125 cm off the ground. Calculate the increase in gravitational potential energy of the baby, taking g as 9.80 N kg^{-1} and giving your answer correct to two significant figures.	
Thinking	Working
Recall the formula for gravitational potential energy.	$E_g = mg\Delta h$
Identify the relevant values for this situation. Subtract the baby's original position off the ground from its final position.	$m = 6\text{ kg}$ $g = 9.80\text{ N kg}^{-1}$ $\Delta h = 125 - 70 = 55\text{ cm} = 0.55\text{ m}$
Substitute the values for this situation into the equation.	$E_g = 6 \times 9.80 \times 0.55$
State the answer with appropriate units and significant figures.	$E_g = 32\text{ J}$

Section 9.4 Review

KEY QUESTIONS SOLUTIONS

- $E_g = mg\Delta h = 0.057 \times 9.80 \times 8.2 = 4.6\text{ J}$
 - $E_g = mg\Delta h = 0.057 \times 9.80 \times 4.1 = 2.3\text{ J}$
- $E_g = mg\Delta h$
 $= 65 \times 9.80 \times (8848 - 5150)$
 $= 2.36 \times 10^6\text{ J}$
 $= 2360\text{ kJ}$
- $E_g = mg\Delta h$
 $20000 = 90.0 \times g \times 60.0$
 $g = \frac{20000}{90.0 \times 60.0}$
 $= 3.70\text{ m s}^{-2}$
- $E_g = mg\Delta h$
 $= 50.0 \times 9.80 \times (1.80 - 0.75)$
 $= 514.5\text{ J}$

So Isabella has jumped with more energy than is required to equal the change in gravitational potential energy. She will therefore clear the bar.

$$\begin{aligned}
 5 \quad E_g &= mg\Delta h \\
 &= 7.50 \times 9.80 \times (-150) \\
 &= -11025 \text{ J}
 \end{aligned}$$

So the eagle's potential energy has decreased by $1.10 \times 10^3 \text{ J}$.

- 6 No. In physics, work against gravity is defined as force exerted over a displacement. Holding the weight above your head might require effort and energy, but you will not be doing any actual work against gravity.

Section 9.5 Law of conservation of energy

Worked example: Try yourself 9.5.1

MECHANICAL ENERGY OF A FALLING OBJECT

A 6.8 kg bowling ball is dropped from a height of 0.75 m. Calculate its kinetic energy at the instant before it hits the ground.	
Thinking	Working
Since the ball is dropped, its initial kinetic energy is zero.	$(E_k)_{\text{initial}} = 0 \text{ J}$
Calculate the initial gravitational potential energy of the ball.	$ \begin{aligned} (E_g)_{\text{initial}} &= mgh \\ &= 6.8 \times 9.80 \times 0.75 \\ &= 50 \text{ J} \end{aligned} $
Calculate the initial mechanical energy.	$ \begin{aligned} (E_m)_{\text{initial}} &= (E_k)_{\text{initial}} + (E_g)_{\text{initial}} \\ &= 0 + 50 \\ &= 50 \text{ J} \end{aligned} $
At the instant the ball hits the ground, its gravitational potential energy is zero.	$(E_g)_{\text{final}} = 0 \text{ J}$
Mechanical energy is conserved in this situation.	$ \begin{aligned} \therefore (E_m)_{\text{initial}} &= (E_m)_{\text{final}} = 50 \\ &= (E_k)_{\text{final}} + 0 \\ (E_k)_{\text{final}} &= 50 \text{ J} \end{aligned} $

Worked example: Try yourself 9.5.2

FINAL VELOCITY OF A FALLING OBJECT

A 6.8 kg bowling ball is dropped from a height of 0.75 m. Calculate its speed at the instant before it hits the ground.	
Thinking	Working
Recall the velocity of the falling object formula.	$v = \sqrt{2gh}$
Substitute the relevant values into the formula and solve.	$ \begin{aligned} v &= \sqrt{2 \times 9.80 \times 0.75} \\ &= 3.8 \text{ m s}^{-1} \end{aligned} $
Interpret the answer.	The bowling ball will be falling at 3.8 m s^{-1} just before it hits the ground.

Worked example: Try yourself 9.5.3
USING MECHANICAL ENERGY TO ANALYSE PROJECTILE MOTION

An arrow with a mass of 35 g is fired into the air at 80 ms^{-1} from a height of 1.4 m. Calculate the speed of the arrow when it has reached a height of 30 m.	
Thinking	Working
Recall the formula for mechanical energy.	$E_m = E_k + E_g = \frac{1}{2}mv^2 + mgh$
Substitute in the values for the arrow as it is fired.	$\begin{aligned}(E_m)_{\text{initial}} &= (E_k)_{\text{initial}} + (E_g)_{\text{initial}} \\ &= \frac{1}{2}mv^2 + mgh \\ &= \frac{1}{2}(0.035 \times 80^2) + (0.035 \times 9.80 \times 1.4) \\ &= 112.5 \text{ J}\end{aligned}$
Use conservation of mechanical energy to find an equation for the final speed.	$\begin{aligned}(E_m)_{\text{final}} &= (E_k)_{\text{final}} + (E_g)_{\text{final}} \\ &= \frac{1}{2}mv^2 + mgh \\ 112.5 &= \frac{1}{2}(0.035)v^2 + (0.035 \times 9.80 \times 30)\end{aligned}$
Solve the equation algebraically to find the final speed.	$\begin{aligned}112.5 &= 0.0175v^2 + 10.3 \\ 102.2 &= 0.0175v^2 \\ v^2 &= \frac{102.2}{0.0175} \\ v &= \sqrt{5840} \\ &= 76 \text{ ms}^{-1}\end{aligned}$
Interpret the answer.	The arrow will be moving at 76 ms^{-1} when it reaches a height of 30 m.

Worked example: Try yourself 9.5.4
ENERGY EFFICIENCY

An electric kettle uses 23.3 kJ of electrical energy as it boils a quantity of water. The efficiency of the kettle is 18%.

How much electrical energy is expended in actually boiling the water?	
Thinking	Working
Recall the equation for efficiency. Substitute the given values into the equation.	$\begin{aligned}\text{efficiency} &= 18\% \\ \text{input} &= 23.3 \text{ kJ} \\ \text{output} &= ? \\ \text{efficiency } (\eta) &= \frac{\text{energy output}}{\text{energy input}} \times 100\% \\ 18 &= \frac{\text{energy output}}{23.3} \times 100\%\end{aligned}$
Solve the equation.	output = 4.19 kJ

Section 9.5 Review

KEY QUESTIONS SOLUTIONS

- 1 a $(E_k)_{\text{final}} = (E_g)_{\text{initial}} = mg\Delta h = 180 \times 9.80 \times 15 = 26\,460\text{J}$
 To two significant figures: 26 000 J
- b $(E_g)_{\text{initial}} = +(E_k)_{\text{final}}$
 $26\,460 = (E_k)_{\text{final}} + mg\Delta h$
 $26\,460 = (E_k)_{\text{final}} + (180 \times 9.80 \times 5)$
 $26\,460 - 8820 = (E_k)_{\text{final}}$
 $(E_k)_{\text{final}} = 17\,640\text{J}$
- 2 a $E_{\text{initial}} = E_{\text{final}}$
 $\frac{1}{2}mv^2 = mg\Delta h$
 $v = \sqrt{2gh} = \sqrt{2 \times 9.80 \times 15} = 17\text{ms}^{-1}$
- b $v = \sqrt{2gh} = \sqrt{2 \times 9.80 \times 10} = 14\text{ms}^{-1}$
- 3 $v = \sqrt{2gh}$
 $h = \frac{v^2}{2g} = \frac{5.4^2}{2 \times 9.80} = 1.5\text{m}$
- 4 a $E_m = E_k + E_g$
 $= mv^2 + mgh$
 $= \frac{1}{2} \times 0.800 \times 28.5^2 + 0.800 \times 9.80 \times 1.45$
 $= 336\text{J}$
- b $E_m = E_k + E_g$
 $336 = \frac{1}{2} \times 0.800 \times v^2 + 0$
 $v = \sqrt{\frac{336}{0.400}}$
 $= 29.0\text{ms}^{-1}$
- 5 Energy output = $\frac{\text{efficiency}}{100} \times \text{energy input}$
 $= \frac{30}{100} \times 2000 = 600\text{J}$
- 6 80% of its E_g is retained so 80% of its height is retained. 80% of 1.5 m = 1.2 m
- 7 The ball is moving upwards, hence its gravitational potential energy is increasing. By the conservation of energy, its kinetic energy must therefore be decreasing by the exact same amount.

Section 9.6 Power

Worked example: Try yourself 9.6.1

CALCULATING POWER

Calculate the power used by a weightlifter to lift a barbell which has a total mass of 50 kg from the floor to a height of 2.0 m above the ground in 1.4 s. (Use $g = 9.80 \text{ m s}^{-2}$.)	
Thinking	Working
Calculate the force applied.	$F_g = mg$ $= 50 \times 9.80$ $= 490 \text{ N}$
Calculate the work done.	$W = F_s$ $= 490 \times 2.0$ $= 980 \text{ J}$
Recall the formula for power.	$P = \frac{W}{\Delta t} \text{ or } P = \frac{\Delta E}{\Delta t}$
Substitute the appropriate values into the formula.	$P = \frac{980}{1.4}$
Solve.	$P = 700 \text{ W}$

Worked example: Try yourself 9.6.2

FORCE-VELOCITY FORMULATION OF POWER

Calculate the power required to keep a car moving at an average speed of 22 m s^{-1} if the force of friction (including air resistance) is 1200 N. Give your answer correct to three significant figures.	
Thinking	Working
Recall the force-velocity formulation of the power equation.	$P = Fv_{av}$
Substitute the appropriate values into the formula.	$P = 1200 \times 22$
Solve.	$P = 26\,400 \text{ W}$

Section 9.6 Review

KEY QUESTIONS SOLUTIONS

- $100 \text{ km h}^{-1} \div 3.6 = 27.8 \text{ m s}^{-1}$

$$P = \frac{\Delta E}{\Delta t}$$

$$= \frac{\frac{1}{2}(1610 \times 27.8^2)}{5.50}$$

$$= 113\,115 \text{ W or } 113 \text{ kW}$$
- $P = 4000 \times 20$

$$= 80\,000 \text{ W}$$

$$= 80 \text{ kW}$$
- $80 \text{ km h}^{-1} \div 3.6 = 22.2 \text{ m s}^{-1}$

$$P = Fv_{av}$$

$$\therefore F = \frac{P}{v_{av}}$$

$$= \frac{40\,000}{22}$$

$$= 1800 \text{ N}$$

$$4 \quad P = Fv_{av}$$

$$\therefore v_{av} = \frac{P}{F}$$

$$F = mg = 500 \times 9.80 = 4900 \text{ N}$$

$$v_{av} = \frac{P}{F} = \frac{25\,000}{4900} = 5.1 \text{ ms}^{-1}$$

$$5 \quad v_{av} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{20.0}{10.0}$$

$$= 2.00 \text{ ms}^{-1}$$

$$P = Fv_{av}$$

$$= 15.0 \times 2.00$$

$$v_{av} = 30.0 \text{ W}$$

$$6 \quad P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$W = mg\Delta h$$

$$= 40.0 \times 9.80 \times 1.50$$

$$= 588 \text{ J}$$

$$\therefore P = \frac{588}{10.0}$$

$$= 58.8 \text{ W}$$

$$7 \quad P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2} \times 800.0 \times (55.0^2 - 40.0^2)$$

$$= 5.70 \times 10^5 \text{ J}$$

$$\therefore 120 \times 10^3 = \frac{5.70 \times 10^5}{\Delta t}$$

$$\therefore \Delta t = \frac{5.70 \times 10^5}{120 \times 10^3}$$

$$= 4.75 \text{ s}$$

CHAPTER 9 REVIEW

- 1 $W = Fs = 2000 \times 80 = 160\,000 \text{ J}$
- 2 Approximately 40 squares, so 40 J
- 3 $W = Fs$
 $= mg \times h$
 $= 200 \times 9.80 \times 30$
 $= 58\,800 \text{ J}$
- 4 For each step: $W = Fs = 60 \times 9.80 \times 0.165 = 97 \text{ J}$
 For all 12 steps: $W = 12 \times 97 = 1200 \text{ J}$
- 5 $F = mg = 50 \times 9.80 = 490 \text{ N}$
 $s = \frac{W}{F} = \frac{4000}{490} = 8.2 \text{ m}$
- 6 $W = Fs \cos \theta$
 $1200 = F \times 20 \cos 35^\circ$
 $F = 73.25 \text{ N}$

$$7 \quad 150 \text{ km h}^{-1} = 42 \text{ m s}^{-1}$$

$$160 \text{ g} = 0.16 \text{ kg}$$

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 0.16 \times 42^2 = 140 \text{ J}$$

$$8 \quad v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 70\,000}{1200}} = 11 \text{ m s}^{-1}$$

$$9 \quad E_k = \frac{1}{2}mv^2 \therefore E_k \propto v^2$$

So doubling the velocity causes E_k to change by a factor of 2^2 or 4.

$$10 \quad E_g = mg\Delta h = 88 \times 9.80 \times 0.40 = 340 \text{ J}$$

$$11 \quad E_m = E_k + E_g$$

$$= \frac{1}{2}mv^2 + mgh$$

$$= \frac{1}{2} \times (0.43 \times 16^2) + (0.43 \times 9.80 \times 0)$$

$$= 55 \text{ J}$$

$$E_m = E_k + E_g$$

$$55 = \frac{1}{2} \times (0.43 \times v^2) + (0.43 \times 9.80 \times 2.44)$$

$$= 0.215v^2 + 10.3$$

$$v = \sqrt{\frac{44.7}{0.215}}$$

$$= 14.4 \text{ m s}^{-1}$$

12 Initial kinetic energy is contained only within the white ball.

So

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 0.160 \times 5.00^2$$

$$= 2.00 \text{ J}$$

$$\text{Final kinetic energy} = \frac{1}{2} \times 0.160 \times 1.00^2 + \frac{1}{2} \times 0.160 \times 4.00^2$$

$$= 1.36 \text{ J}$$

The final kinetic energy of the system is lower, hence the collision was inelastic.

$$13 \text{ a } E_g = mgh$$

$$= 1.51 \times 9.80 \times 0.15 = 2.2 \text{ J}$$

b The gain in gravitational potential energy of the pendulum (2.2 J) is equal to the kinetic energy of the pendulum as it starts to swing upwards, so the pendulum had 2.2 J of kinetic energy.

$$\text{c } v = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{(2 \times 2.2 \div 1.51)}$$

$$= 1.7 \text{ m s}^{-1}$$

14 Remember: $P = \frac{W}{\Delta t}$ and $1 \text{ kW} = 1000 \text{ W}$

$$P = \frac{mgh}{\Delta t}$$

$$= \frac{5000 \times 9.80 \times 20}{5}$$

$$= 196\,000 \text{ W}$$

$$= 196 \text{ kW} \approx 200 \text{ kW}$$

$$\begin{aligned} 15 \quad \Delta E_k &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times (650 \times 27.8^2) - 0 \\ &= 251\,000\text{J} \\ &= 251\text{kJ} \end{aligned}$$

$$\begin{aligned} P &= \frac{\Delta E}{t} \\ &= \frac{251}{7.2} \\ &= 35\text{kW} \end{aligned}$$

$$16 \quad F = \frac{P}{V_{av}} = \frac{25\,000}{17} = 1500\text{N}$$

$$17 \quad \text{a} \quad W = E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 60 \times 8^2 = 1920\text{J}$$

$$\text{b} \quad F = \frac{W}{s} = \frac{1920}{20} = 96\text{N}$$

$$18 \quad E_g = mg\Delta h = 120 \times 1.6 \times 0.1 = 19.2\text{J}$$

$$19 \quad \text{Efficiency } (\eta) = \frac{\text{energy output}}{\text{energy input}} \times 100\%$$

$$80 = \frac{1250}{\text{input}} \times 100$$

$$\text{input} = \frac{1250}{80} \times 100$$

$$= 1562.5\text{J}$$

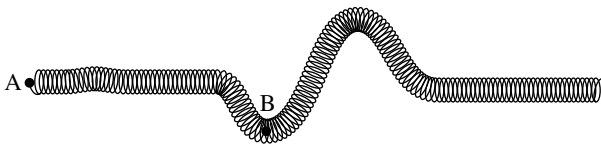
Chapter 10 The nature of waves

Section 10.1 Longitudinal and transverse waves

Section 10.1 Review

KEY QUESTIONS SOLUTIONS

- The particles oscillate back and forth or up and down around a central or average position and pass on the energy carried by the wave. They do not move along with the wave.
- False: Longitudinal waves occur when particles vibrate backwards and forwards about a mean position, parallel to the direction of the wave.
 - True.
 - True.
 - True.
- The pulse will move to the right and point B will move downwards.



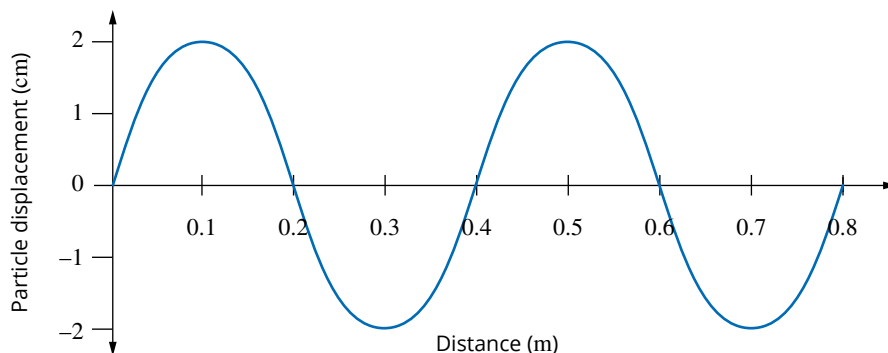
- Mechanical waves: sound, ripples on a pond, vibrations in a rope. (Light does not require the particles of a medium to propagate and is therefore not a mechanical wave.)
- The tuning fork vibrates back and forth, creating a series of compressions and rarefactions in the air as the energy is transferred.
- The forward motion of the source (for example, the speaker or tuning fork) pushes particles together so particle A goes to the right, the backward motion of the source creates a low-pressure area (the rarefaction) as particle B is moved to the left.
- In a transverse wave the motion of the particles is at right angles (perpendicular) to the direction of travel of the wave itself.
- Longitudinal: a and d
Transverse: b and c

Section 10.2 Representing waves

Worked example: Try yourself 10.2.1

DISPLACEMENT-DISTANCE GRAPHS 1

The displacement–distance graph below shows a snapshot of a transverse wave as it travels along a spring towards the right. Use the graph to determine the wavelength and the amplitude of this wave.

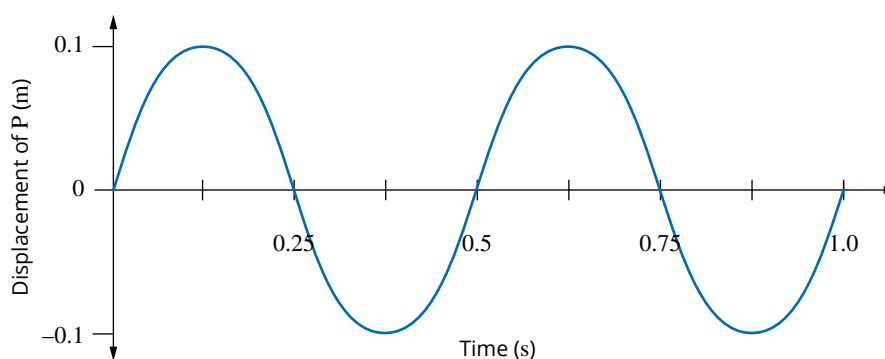


Thinking	Working
Amplitude on a displacement–distance graph is the distance from the average position to a crest or trough.	Amplitude = 2 cm = 0.02 m
Wavelength is the distance for one complete cycle. Any two consecutive points in phase and at the same position on the wave could be used.	Wavelength, $\lambda = 0.4$ m

Worked example: Try yourself 10.2.2

DISPLACEMENT–TIME GRAPHS 2

The displacement–time graph below shows the motion of a single part of a rope as a wave passes, travelling to the right. Use the graph to find the amplitude, period and frequency of the wave.



Thinking	Working
Amplitude on a displacement–time graph is the displacement from the average position to a crest or trough. Note the displacement of successive crests and/or troughs on the wave and carefully note units on the vertical axis.	Amplitude = 0.1 m
Period is the time it takes to complete one cycle and can be identified on a displacement–time graph as the time between two successive points that are in phase. Identify two points on the graph at the same position in the wave cycle. Confirm by checking two other points, e.g. two crests or two troughs.	Period, $T = 0.5$ s
Frequency can be calculated using $f = \frac{1}{T}$, measured in hertz (Hz).	$f = \frac{1}{T} = \frac{1}{0.5} = 2$ The frequency is 2 Hz.

Worked example: Try yourself 10.2.3

THE WAVE EQUATION 1

A longitudinal wave has a wavelength of 3.00 m and a speed of 1484 ms^{-1} . What is the frequency, f , of the wave?

Thinking	Working
The wave equation states that $v = f\lambda$. Knowing both v and λ , the frequency, f , can be found. Rewrite the wave equation in terms of f .	$v = f\lambda$ $f = \frac{v}{\lambda}$
Substitute the known values and solve.	$f = \frac{v}{\lambda}$ $= \frac{1484}{3.00}$ $= 495 \text{ Hz}$

Worked example: Try yourself 10.2.4
THE WAVE EQUATION 2

A longitudinal wave has a wavelength of 3.00 m and a speed of 1484 ms ⁻¹ . Calculate the period, T , of the wave.	
Thinking	Working
Rewrite the wave equation in terms of T .	$v = f\lambda, \text{ and } f = \frac{1}{T}$ $v = \frac{1}{T} \times \lambda$ $= \frac{\lambda}{T}$ $T = \frac{\lambda}{v}$
Substitute the known values and solve.	$T = \frac{\lambda}{v}$ $= \frac{3.00}{1484}$ $= 0.00202 \text{ s}$ $= 2.02 \times 10^{-3} \text{ s}$

Section 10.2 Review

KEY QUESTIONS SOLUTIONS

- C and F
 - wavelength
 - B and D
 - amplitude
- Wavelength is the length of one complete wave cycle. Any two points at the same position on the wave could be used. In this case $\lambda = 1.6 \text{ m}$.

Amplitude is the displacement from the average position to a crest or trough. In this case, amplitude = 20 cm.
- period = 0.4 s
 - $f = \frac{1}{T} = \frac{1}{0.4} = 2.5 \text{ Hz}$
- $f = 5 \text{ Hz}$, amplitude = 0.3 m, $\lambda = 1.3 \text{ m}$, $v = ?$

$$v = f\lambda = 5 \times 1.3 = 6.5 \text{ ms}^{-1}$$
- True.
 - False: The period of a wave is *proportional* to its wavelength.
 - True.
 - False: The wavelength *and* frequency of a wave determine its speed.
- wavelength = 4 cm; amplitude = 0.5 cm
 - $T = 2 \text{ s}$, $\lambda = 4 \text{ cm}$, $v = ?$
$$v = \frac{\lambda}{T} = \frac{4}{2} = 2 \text{ cm s}^{-1} \text{ or } 0.02 \text{ ms}^{-1}$$
 - red
- $$T = \frac{1}{f} = \frac{1}{2 \times 10^5} = 5 \times 10^{-6} \text{ s}$$
- As the speed of each vehicle is the same and there is no relative motion of the medium, the frequency observed would be the same as that at the source.
- The apparent frequency increases when travelling towards you, so the siren would sound higher in pitch, and decreases when travelling away from you, so the siren would sound lower.

Section 10.3 Wave behaviours—reflection, refraction and diffraction

Worked example: Try yourself 10.3.1

REFRACTION

Whales typically emit sounds between 10 and 40 Hz (humans can usually hear down to 20 Hz). If a whale emits a 20 Hz sound in water towards the surface at an angle of 40° to the normal, the refracted wave emerges from the water into air. The speed of sound in air is 343 ms^{-1} and the speed of sound in water is 1484 ms^{-1} .

a Find the wavelength in water and in air.	
Thinking	Working
The wavelength can be determined from $v = f\lambda$. Rearrange to make λ the subject: $\lambda = \frac{v}{f}$ where $f = 40 \text{ Hz}$ $v_{\text{air}} = 343 \text{ ms}^{-1}$ $v_{\text{water}} = 1484 \text{ ms}^{-1}$	$\lambda_{\text{water}} = \frac{1484}{40} = 37.1 \text{ m}$ $\lambda_{\text{air}} = \frac{343}{40} = 8.58 \text{ m}$
b Explain what will happen to the refracted wave, and why.	
The speed of sound in air is much slower than in water.	Since the speed in air is lower than in water, the angle will refract significantly towards the normal.
c Determine the angle of refraction.	
The angle of refraction can be calculated using Snell's law. $\frac{\sin r}{\sin i} = \frac{v_{\text{air}}}{v_{\text{water}}}$	$\sin r = \frac{v_{\text{air}}}{v_{\text{water}}} \sin i$ $\sin r = \frac{343}{1484} \sin 40^\circ$ $= 0.149$ $r = 8.6^\circ$

Worked example: Try yourself 10.3.2

TOTAL INTERNAL REFLECTION

Sound is travelling through air and hits a steel wall. At what angle is the sound totally reflected? The speed of sound in steel is 5000 ms^{-1} and the speed of sound in air is 340 ms^{-1} .	
Thinking	Working
The critical angle needs to be calculated using Snell's law $\frac{\sin r}{\sin i} = \frac{v_2}{v_1}$ Set $\sin r = 1$ $v_1 = 340 \text{ ms}^{-1}$ $v_2 = 5000 \text{ ms}^{-1}$	$\sin \theta_c = \frac{v_1}{v_2}$ $\sin \theta_c = \frac{340}{5000} = 0.068$ $\theta_c = 3.99^\circ$

Worked example: Try yourself 10.3.3
DIFFRACTION

In ultrasound imaging, the speed of sound is 1540 m s^{-1} . The resolution of an image depends on the wavelength of the sound—a smaller wavelength (higher frequency) enables more detail to be seen with less effect of diffraction. High-frequency sound (5 to 10 MHz) can resolve more detail but has limited penetration depth, whereas low-frequency sound (2 to 5 MHz) can penetrate to deeper structures but has lower resolution.

a If the human heart is 10 cm across, what frequency is needed to have at least 300 wavelengths across the image?	
Thinking	Working
The human heart is 10 cm across. 300 wavelengths need to fit into 10 cm.	$\lambda = \frac{10 \times 10^{-2}}{300}$ $= 0.000333 \text{ m}$
Calculate the frequency using: $f = \frac{v}{\lambda}$	$f = \frac{1540}{0.000333}$ $= 4.67 \times 10^6 \text{ Hz}$ $= 4.67 \text{ MHz}$
b If the frequency were significantly lower than your calculated amount, what would happen to the image? Explain why.	
As the frequency gets lower, the wavelength gets longer.	As the frequency gets lower, the image would be harder to resolve as diffraction effects would become greater.

Section 10.3 Review

KEY QUESTIONS SOLUTIONS

- The wave is reflected and there is a 180° change in phase.
- As a wave is reflected back into the same medium, the only property that will change is amplitude. This is because some of the energy of the wave has been absorbed by the second medium from which the wave was reflected. (Note: the velocity will change as the wave changes direction, but its speed will not change because it is a scalar quantity.)
- C. The object must be convex, that is, curved outwards.
- B. Angles are labelled relative to the normal.
- After an earthquake, P waves (longitudinal) can be detected after travelling through the region and can be refracted, but S waves (transverse) are not detected. P waves can travel through a fluid but S waves cannot. P waves are also refracted as they travel through the liquid interior and when they cross a boundary between different layers.
- B, C and D. Since refraction occurs travelling from deep water to shallow, it indicates there is a change in speed (D) which will cause a change in wavelength (B) and result in a change in angle or direction (C). The frequency of the wave (A) will remain unchanged.
- As wheel B rolls onto the grass it *slows down*. Since wheel A is now moving *faster* than wheel B, the wheels change *direction*. When wheel A rolls onto the grass the wheels' direction *stops* changing.
- $v_{20} = 331 + 0.60T = 331 + 0.60 \times 20 = 343 \text{ m s}^{-1}$
 - $v_{30} = 331 + 0.60T = 331 + 0.60 \times 30 = 349 \text{ m s}^{-1}$
 - The speed increases therefore the refracted angle will increase.
 - $\frac{\sin r}{\sin i} = \frac{v_{30}}{v_{20}}$ so $\sin r = \frac{v_{30} \times \sin i}{v_{20}} = \frac{349 \times \sin 50}{343} = 0.779$ and $r = 51.2^\circ$
 - Total internal reflection $\sin i = \frac{v_{20}}{v_{30}} = \frac{343}{349} = 0.983$, so $i = 79.4^\circ$. Therefore the angle must be greater than 79° .
- The higher frequency sound of the flute corresponds to a shorter wavelength so it will be diffracted less and will be more directional. Therefore, it will not be heard as well at the sides of the auditorium. The tuba undergoes a lot more diffraction and so will be louder at the sides.

Section 10.4 Wave interactions—superposition, interference and resonance

Section 10.4 Review

KEY QUESTIONS SOLUTIONS

- True.
 - False: As the pulses pass through each other, the interaction *does not* permanently alter the characteristics of each pulse.
 - True.
- B. Each pulse travels 3 m in 3 s. Adding their amplitudes together means they will look like C, but the result is they will cancel each other out as in B.
- An object subjected to forces with a forcing frequency matching its natural oscillating frequency will oscillate with increasing amplitude as there is a maximum transfer of energy. This could continue until the structure can no longer withstand the internal forces and fails.
- $\theta_i = 90^\circ - 38^\circ = 52^\circ$
 $\theta_r = \theta_i = 52^\circ$
- Normal walking results in a frequency of 1 Hz or 1 cycle per second i.e. two steps per second. This frequency may result in an increase in the amplitude of oscillation of the bridge over time, which could damage the structure.

Section 10.5 Standing waves and harmonics

Worked example: Try yourself 10.5.1

FUNDAMENTAL FREQUENCY

A standing wave in a string is found to have a wavelength of 0.50 m for the fundamental frequency of vibration. Assume that the tension in the string is not changed and that the string is fixed at both ends.

a Calculate the length of the string.	
Thinking	Working
Identify the wavelength of the string (λ) in metres and the harmonic number (n).	$\lambda = 0.5 \text{ m}$ $n = 1$
Recall that for any frequency, $\lambda = \frac{2\ell}{n}$. Rearrange to find ℓ .	$\lambda = \frac{2\ell}{n}$ $\ell = \frac{n\lambda}{2}$
Substitute the value for the wavelength from the question and solve for ℓ .	$\ell = \frac{1 \times 0.5}{2}$ $= 0.25 \text{ m}$

b Calculate the wavelength of the third harmonic.	
Thinking	Working
Identify length of the string (ℓ) in metres and the harmonic number (n)	$\ell = 0.25 \text{ m}$ $n = 3$
Recall that for any frequency $\lambda = \frac{2\ell}{n}$.	$\lambda = \frac{2\ell}{n}$
Substitute the values from the question and solve for λ .	$= \frac{2 \times 0.25}{3}$ $= 0.17 \text{ m}$

Worked example: Try yourself 10.5.2
OPEN-ENDED AIR COLUMNS

The wavelength of the fourth harmonic in a tube that can be considered as an open-ended air column is found to be 12 cm.

a Calculate the length of the tube, assuming that the standing wave does not extend beyond the ends of the tube.	
Thinking	Working
Identify the wavelength of the sound (λ) in metres and the harmonic number (n).	$\lambda = 0.12 \text{ m}$ $n = 4$
Recall that for any frequency, $\lambda = \frac{2\ell}{n}$ Rearrange to find ℓ .	$\ell = \frac{n\lambda_n}{2}$ $= \frac{4 \times 0.12}{2}$ $= 0.24 \text{ m}$

b Determine the fundamental frequency.	
Thinking	Working
Recall for the fundamental frequency half a wavelength fits into the length of the pipe. $\ell = \frac{\lambda_1}{2}$	$\lambda_1 = 2\ell$ $= 2 \times 0.24$ $= 0.48 \text{ m}$
Then $f_1 = \frac{v}{\lambda_1}$	$f_1 = \frac{340}{0.48}$ $= 708 \text{ Hz}$

Worked example: Try yourself 10.5.3
AIR COLUMN CLOSED AT ONE END

An air column closed at one end is 12 cm long. Assume that the standing wave does not extend beyond the end of the tube.

a Calculate the wavelength of the fifth harmonic.	
Thinking	Working
Identify the length of the air column (ℓ) in metres and the harmonic number (n).	$\ell = 20 \text{ cm}$ $n = 5$
Recall that $\lambda_n = \frac{4\ell}{n}$ Substitute the values from the question and solve for λ .	$\lambda_5 = \frac{4\ell}{5}$ $= \frac{4 \times 0.20}{5}$ $= 0.16 \text{ m}$

b Calculate the frequency of the fifth harmonic if the velocity of sound is 340 ms^{-1} .	
Thinking	Working
Identify the speed of the sound (v) in ms^{-1} and the wavelength (λ) from the previous question.	$v = 340 \text{ ms}^{-1}$ $\lambda_5 = 0.16 \text{ m}$
Recall the wave equation $v = f\lambda$. Rearrange to find f .	$f_5 = \frac{v}{\lambda_5}$ $= \frac{340}{0.16}$ $= 2130 \text{ Hz}$

Section 10.5 Review

KEY QUESTIONS SOLUTIONS

- 1 It is a common misconception that standing waves somehow remain stationary. It is only the pattern made by the amplitude along the rope that stays still at the nodes. The rope is still moving, especially at the antinodes.
- 2 A transverse wave moving along a slinky spring is reflected at a fixed end with a phase change. The interference that occurs during the superposition of this reflected wave and the original wave creates a standing wave. This standing wave consists of locations called nodes, where the movement of the spring is cancelled out, and antinodes where maximum movement of the spring occurs. Nodes always occur at the ends.
- 3 $\lambda = \frac{2\ell}{n} = \frac{2 \times 0.4}{1} = 0.8 \text{ m}$
- 4 Rearranging $\lambda = \frac{2\ell}{n}$ gives $\ell = \frac{n\lambda}{2} = \frac{4 \times 0.75}{2} = 1.5 \text{ m}$
- 5 This wave will have a frequency four times that of the fundamental frequency, which means that it will have a wavelength $\frac{1}{4}$ of the fundamental wavelength due to the inverse relationship between frequency and wavelength.
- 6 The wavelength of the standing wave in the diagram is 5 m. The wavelength of the fundamental frequency is twice the length of the string. Therefore, a string length of 2.5 m would produce a standing wave with wavelength 5 m.
- 7 $f = \frac{nv}{2\ell}$
 $\ell = \frac{nv}{2f} = \frac{1 \times 387}{2 \times 350} = 0.55 \text{ m}$
 new length = $\frac{2}{3} \times 0.55 = 0.37 \text{ m}$
 new wavelength = $2 \times \text{new length} = 2 \times 0.37 = 0.74 \text{ m}$
- 8 **a** $f = \frac{nv}{2\ell} = \frac{1 \times 300}{2 \times 0.5} = 300 \text{ Hz}$
b $f = \frac{nv}{2\ell} = \frac{2 \times 300}{2 \times 0.5} = 600 \text{ Hz}$
 Or use $f_2 = 2f_1 = 2 \times 300 = 600 \text{ Hz}$
c $f = \frac{nv}{2\ell} = \frac{3 \times 300}{2 \times 0.5} = 900 \text{ Hz}$
 Or use $f_3 = 3f_1 = 3 \times 300 = 900 \text{ Hz}$
- 9 Open-ended pipe, so use $\lambda_n = \frac{2\ell}{n}$
a $n = 1$, so $\lambda_1 = \frac{2\ell}{1} = \frac{2 \times 0.450}{1} = 0.900 \text{ m}$
b $n = 2$, so $\lambda_2 = \frac{2\ell}{2} = \frac{2 \times 0.450}{2} = 0.450 \text{ m}$
c $n = 3$, so $\lambda_2 = \frac{2\ell}{3} = \frac{2 \times 0.450}{3} = 0.300 \text{ m}$, and from $v = f\lambda$ rearrange:
 $f_3 = \frac{v}{\lambda_3} = \frac{330.0}{0.300} = 1100 \text{ Hz}$
- 10 The pipe is closed at one end, therefore use $f_n = \frac{nv}{4\ell}$
a Using $f_n = \frac{nv}{4\ell}$ where n is the harmonic number,
 $n = 1$ so $f_1 = \frac{1 \times v}{4\ell} = \frac{1 \times 330}{4 \times 0.750} = 110 \text{ Hz}$
b $n = 3$ so $f_3 = \frac{3 \times v}{4\ell} = \frac{3 \times 330}{4 \times 0.750} = 330 \text{ Hz}$
c Next harmonic along is $n = 5$ and $n = 7$
 So $f_5 = 5 \times f_1 = 550 \text{ Hz}$
 $f_7 = 7 \times f_1 = 770 \text{ Hz}$

- 11 a $f_1 = 450$ Hz. Each of the subsequent frequencies is double the previous, therefore all harmonics are formed and the pipe is open ended.
- b $\lambda_1 = 3.0$ m. The next resonance wavelength is $\frac{1}{3}$ that of the fundamental and the following is $\frac{1}{5}$ of the fundamental. This means the frequencies would be 3 times the fundamental and 5 times the fundamental, therefore the pipe is closed at one end.

Section 10.6 Wave intensity and applications of wave properties

Worked example: Try yourself 10.6.1

INTENSITY AND DISTANCE 1

Sam heard an annoying sound from 100 m away. By what factor would the intensity of the annoying sound change if Sam was to move to a distance of 400 m from the sound?	
Thinking	Working
Intensity, I , will decrease with the square of the distance, r , from the source. The ratio of the intensity at 400 m to the original intensity at 100 m is the factor required. Identify the initial values I_0 and r_0 and the final values I_f and r_f	$r_0 = 100$ m $I_0 \propto \frac{1}{r_0^2}$ then $I_0 \propto \frac{1}{100^2}$ $r_f = 400$ m $I_f \propto \frac{1}{r_f^2}$ then $I_f \propto \frac{1}{400^2}$
Determine the relationship between the intensity and radii.	$\frac{I_f}{I_0} = \frac{r_0^2}{r_f^2}$
Evaluate.	$\frac{I_f}{I_0} = \frac{100^2}{400^2}$ $\frac{I_f}{I_0} = 0.06$

Worked example: Try yourself 10.6.2

INTENSITY AND DISTANCE 2

A fog horn was originally heard from a boat when the boat was 1 km from the fog horn. After some time, the intensity of the fog horn was measured as being half of the original. Assuming the volume of the fog horn hadn't changed, how far away was the boat from the fog horn when the intensity was measured?	
Thinking	Working
Intensity, I , will decrease with the square of the distance, r , from the source. The expression $\frac{I_f}{I_0} = \frac{r_0^2}{r_f^2}$ can be used. Identify the variables r_0 and $\frac{I_f}{I_0}$.	$r_0 = 1000$ m $\frac{I_f}{I_0} = \frac{1}{2}$
Rearrange the expression and evaluate.	$\frac{I_f}{I_0} = \frac{r_0^2}{r_f^2}$ $r_f^2 = \frac{r_0^2 I_0}{I_f}$ $r_f^2 = \frac{1000^2}{\frac{1}{2}} = 1000^2 \times 2 = 2 \times 10^6$ $r_f = 141$ m

Section 10.6 Review

KEY QUESTIONS SOLUTIONS

- $I_0 = 2.5 \times 10^4 \text{ W m}^{-2}$, $I_0 \propto \frac{1}{r_0^2} = \frac{1}{250^2}$ and $I_f \propto \frac{1}{r_f^2} = \frac{1}{1000^2}$, therefore $I_f = 2.5 \times 10^4 \times \frac{250^2}{1000^2} = 1560 \text{ W m}^{-2}$
- $r_0 = 20 \text{ m}$, $\frac{I_f}{I_0} = \frac{1}{8}$ then $\frac{I_f}{I_0} = \frac{r_0^2}{r_f^2}$ and $r_f^2 = \frac{r_0^2}{\frac{I_f}{I_0}} = \frac{20^2}{0.125} = 20^2 \times 8 = 3200$ therefore $r_f = 57 \text{ m}$
- At high-intensity the vibration of the tissues as the ultrasound is propagated produces heat, which can promote faster healing.
- Use an absorber like foam on the ceiling and carpets on the floor.
- A diffuser is a corrugated surface that allows reflections to occur in multiple directions, thus reducing echoes. Standing waves also cannot form.
- A high, solid barrier can be used to reflect the sound; vegetation can be used to absorb the sound.

CHAPTER 10 REVIEW

- The particles on the surface of the water move up and down as the waves radiate outwards, carrying energy away from the point on the surface of the water where the stone entered the water.
- Similarities: both are waves, both carry energy away from the source, both are caused by vibrations.
Differences: transverse waves involve particle displacement at right angles to the direction of travel of the wave; longitudinal waves involve particle displacement parallel to the direction of travel of the wave.
- U is moving down and V is momentarily stationary (and will then move downwards).
- $f = 10.0 \text{ Hz}$, $\lambda = 30.0 \text{ mm} = 0.0300 \text{ m}$, $v = ?$
 $v = f\lambda = 10 \times 0.03 = 0.300 \text{ m s}^{-1}$
- $f = 32\,000$, $v = 1400$, $\lambda = ?$
 $v = f\lambda$ rearranges to $\lambda = \frac{v}{f}$
 $\lambda = 1400 \div 32\,000 = 0.044 \text{ m}$
- C and D. Since the frequency rose and fell, the bike must have travelled past you. It must have come towards you and then moved away from you.
- By inspecting the wave equation $v = f\lambda$, since wavelength decreases and the velocity must stay the same, the frequency must increase. This ensures the product of the wavelength and frequency still equals the velocity, which has remained unchanged. (Note: velocity is constant as it is a property of the medium.)
- The green wave represents the superposition of the blue and the red waves as the amplitude of the green wave is the sum of the amplitudes of the blue and red waves.
- Sound waves are longitudinal mechanical waves where the particles only move back and forth around an equilibrium position, parallel to the direction of travel of the wave. When these particles move in the direction of the wave, they collide with adjacent particles and transfer energy to the particles in front of them. This means that kinetic energy is transferred between particles in the direction of the wave through collisions. Therefore, the particles cannot move along with the wave from the source as they lose their kinetic energy to the particles in front of them during the collisions.
- C and D. Only energy is transferred by a wave therefore the statements saying that air particles have travelled to Lee are incorrect. Energy has been transferred from the speaker to Lee and it is the air particles that have passed this energy along through the air.
- All objects/materials have a resonant frequency. If the object is made to vibrate at this frequency, the amplitude of the object's vibrations will increase with time. If a building or bridge was subjected to wind that made it vibrate at its natural frequency, this vibration may increase in amplitude so much that the structure is damaged or collapses.
- $f = \frac{nv}{4\ell} = \frac{1 \times 340}{4 \times 0.85} = 100 \text{ Hz}$
- $f = \frac{nv}{4\ell} = \frac{3 \times 340}{4 \times 0.85} = 300 \text{ Hz}$

- 14** The fundamental frequency is given by:

$$f_1 = \frac{1}{T} = \frac{1}{4.0} = 0.25 \text{ Hz}$$

The frequency of the second harmonic is given by:

$$f_2 = 2 \times f_1 = 2 \times 0.25 = 0.50 \text{ Hz}$$

- 15** Calculate the wavelength of the wave using the wave equation:

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{78}{428}$$

$$= 0.182 \text{ m}$$

Since the separation of antinodes and of nodes in a standing wave in a string with fixed ends is half the wavelength, then:

$$d = \frac{\lambda}{2} = \frac{0.182}{2}$$

$$= 0.091 \text{ m or } 9.1 \text{ cm}$$

- 16** All of the options are correct. The light rays striking all of these surfaces will obey the law of reflection, as it always holds regardless of the shape of the reflector.

- 17** B. In resonance, maximum energy is transferred and the amplitude of vibration will increase. The frequency is unchanged.

- 18** $\lambda_3 = \frac{v}{f_3} = \frac{330}{400} = 0.825 \text{ m}$; $\lambda_3 = \frac{2\ell}{n}$ so $\ell = \frac{n \times \lambda_3}{2} = \frac{3 \times 0.825}{2} = 1.24 \text{ m}$

- 19 a** The harmonic $n = 1$: $f_1 = \frac{1 \times v}{4\ell} = \frac{1 \times 340}{4 \times 0.85} = 100 \text{ Hz}$

b Third harmonic: $f_3 = \frac{3 \times v}{4\ell} = \frac{3 \times 340}{4 \times 0.85} = 300 \text{ Hz}$, or $f_3 = 3 \times f_1 = 300 \text{ Hz}$

- 20** The angle of refraction from the normal to the refracted ray would decrease relative to the angle of incidence. The speed of sound in air is less than the speed of sound in water, therefore the refracted angle would be smaller.

- 21** Total internal reflection occurs when the wave goes from a slow-speed medium to a higher-speed medium. The refracted angle increases. At the critical angle the angle of refraction is at 90° and lies along the interface between the two media. Any angles greater than this will reflect back into the first medium.

- 22** D. $I_1 = x$, $r_2 = 2r_1$, $I_1 \propto \frac{1}{r_1^2}$ and $I_2 \propto \frac{1}{r_2^2} = \frac{1}{(2r_1)^2} = \frac{1}{4r_1^2}$ therefore $I_2 = \frac{1}{4}I_1 = \frac{x}{4}$

- 23** Increasing the distance by a factor of 4, from 1000 m to 4000 m, reduces the intensity by a factor of 16, from 1 W m^{-2} to $\frac{1}{16} \text{ W m}^{-2}$. Therefore, to collect 1 W the dish would need to have an area of 16 m^2 . Using $A = \pi r^2$,

then $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{16}{\pi}} = 2.26 \text{ m}$, therefore the diameter is 4.514 m (to the nearest mm).

Chapter 11 Practical investigation

Section 11.1 Designing and planning the investigation

11.1 Review

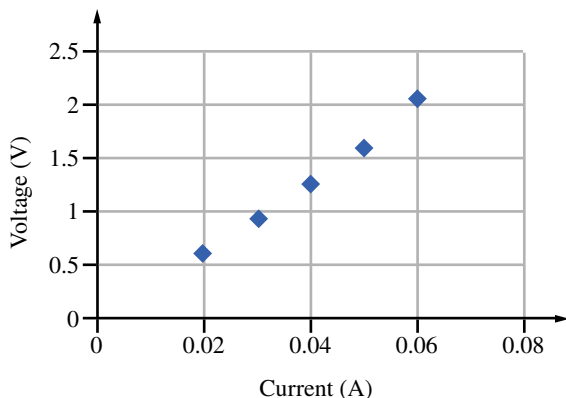
- 1
 - a If the voltage is measured in units of number of batteries then it is a discrete value.
 - b If the voltage is measured with a voltmeter then the voltage would be continuous.
- 2 qualitative
- 3 A. Hypothesis 1 is the best answer as it is a definite statement of the relationship between the independent and dependent variables.
- 4
 - a valid
 - b reliable
 - c accurate
- 5
 - a the tension in the elastic band
 - b the initial launch velocity of the elastic band
 - c the same elastic band, elastic band held in the same way, elastic band launched in the same direction, elastic band placed on the finger in the same way

Section 11.2 Conducting investigations and recording and presenting data

11.2 Review

- 1
 - a systematic error
 - b random error
- 2 Give answer to two significant figures, as this is the least number of significant figures in the data provided.
- 3
 - a $\text{mean} = (21 + 28 + 19 + 19 + 25 + 24) \div 6 = 22.7$
 - b mode = 19
 - c median = 22.5
 - d uncertainty in the mean: $28 - 23 = \pm 5$

4



- 5 as a line of best fit on the graph

Section 11.3 Discussing investigations and drawing evidence-based conclusions

11.3 Review

- 1 A linear graph shows the proportional relationship between two variables.
- 2 an inversely proportional relationship
- 3 a directly proportional relationship
- 4 time restraints and limited resources
- 5 An increase in current from 0.03 to 0.05 A produced an increase of 0.88 V across the resistor.

CHAPTER 11 REVIEW

- 1 A hypothesis is a prediction, based on evidence and prior knowledge, to answer the research question. A hypothesis often takes the form of a proposed relationship between two or more variables.
- 2 Dependent variable: flight displacement
Independent variable: release angle
Controlled variable: (any of) release velocity, release height, landing height, air resistance (including wind)
- 3
 - a the acceleration of the object
 - b the vertical acceleration of the falling object
 - c the rate of rotation of the springboard diver
- 4 Elimination, substitution, isolation, engineering controls, administrative controls, personal protective equipment.
- 5 $6.8 \pm 0.4 \text{ cm s}^{-1}$
- 6 the mean
- 7 an exponential relationship
- 8 This graph should show a straight line with a positive gradient.
- 9 Any issues that could have affected the validity, accuracy, precision or reliability of the data plus any sources of error or uncertainty.
- 10 Bias is a form of systematic error resulting from a researcher's personal preferences or motivations.

Unit 2 REVIEW

1 The weight will travel three times as far during the second second as during the first.

2 a Acceleration = $\frac{\text{resultant force}}{\text{total mass}}$

$$\therefore a = \frac{(20 - 10) \times (9.80)}{30} = 3.3 \text{ ms}^{-2} \text{ clockwise}$$

i.e. 3.3 ms^{-2} up for the 10 kg mass and 3.3 ms^{-2} down for the 20 kg mass.

b The resultant force on the $F_a = F_g + T$

$$ma = mg + T$$

$$20 \times (-3.3) = 20 \times (-9.80) + T$$

$$T = -66 + 196$$

$$T = 1.3 \times 10^2 \text{ N}$$

3 The steady force applied by the engine is equal and opposite to the combined resistance forces such as air resistance and friction between the wheels and track. The net resultant force on the carriages is zero, and according to Newton's first and second laws, constant velocity is the result.

4 a $v^2 = u^2 + 2as$

$$(10.0)^2 = (5.00)^2 + 2a \times (100)$$

$$\therefore a = 0.375 \text{ ms}^{-2} \text{ west}$$

b $\Sigma F = ma = 1000 \times 0.375 \text{ west} = 375 \text{ N west}$

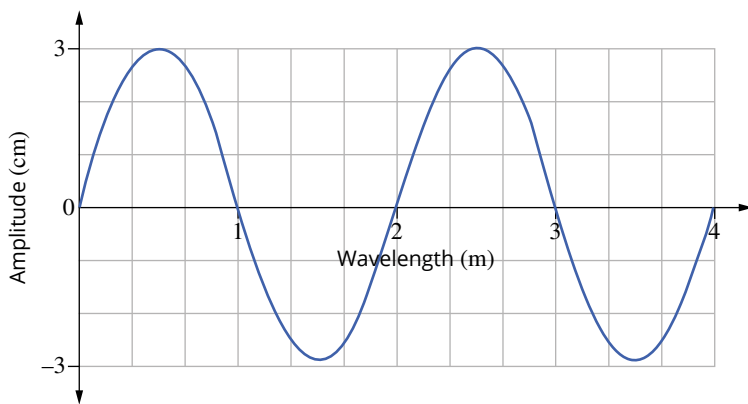
c $\Sigma F =$ force exerted by tow truck – frictional force

$$375 \text{ N} = \text{force exerted by tow truck} - 200 \text{ N}$$

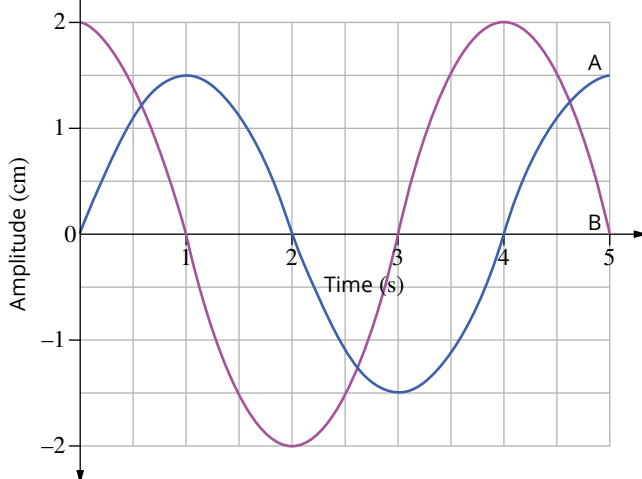
$$\therefore \text{force exerted by tow truck} = 375 \text{ N} + 200 \text{ N} = 575 \text{ N west}$$

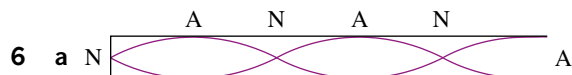
d The tow truck and car form an action–reaction pair, so the car exerts the opposite force: 575 N east.

5 a



b





$$b \quad \lambda = \frac{4l}{(2n-1)} = \frac{4 \times 1.14}{1} = 4.56 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{346}{4.56} = 75.9 \text{ Hz}$$

- 7 The line from Jenny to the speaker in front of her and the line between the speakers make a right angle. Applying Pythagoras' theorem, the distance from Jenny to the other speaker is 3.90 m. The path distance between Jenny and the two speakers is therefore 0.30 m. For constructive interference, the path difference between Jenny and the two speakers must be an odd multiple of half wavelengths.

$$\text{Therefore } \frac{\lambda}{2} = 0.30, \text{ so } \lambda = 0.60 \text{ m}$$

$$\text{or } \frac{3\lambda}{2} = 0.30, \text{ so } \lambda = 0.20 \text{ m}$$

$$\text{or } \frac{5\lambda}{2} = 0.30, \text{ so } \lambda = 0.12 \text{ m}$$

and $f = \frac{v}{\lambda}$, therefore for the first three frequencies, $f = 576.7 \text{ Hz}$ or 1730 Hz or 2883 Hz . (Any multiples of 576.7 Hz are acceptable.)

- 8 a It is a three-dimensional wave that loses energy as it travels through the Earth's crust, with its intensity inversely proportional to the square of the distance travelled.

$$b \quad I \propto \frac{1}{r^2}$$

$$\therefore Ir^2 = \text{constant}$$

$$\therefore I_1(r_1)^2 = I_2(r_2)^2$$

$$I_2 = \frac{I_1(r_1)^2}{(r_2)^2}$$

$$= \frac{(1.0 \times 10^5) \times (100)^2}{(500)^2}$$

$$= 4.0 \times 10^4 \text{ W m}^{-2}$$

- 9 a For the first 30 s, the cyclist travels 150 m east at constant speed, then she accelerates for the next 10 s travelling a further distance of 150 m. She then travels at a higher constant speed for the next 10 s, travelling a further distance of 200 m.

$$b \quad 5 \text{ m s}^{-1} \text{ east}$$

$$c \quad 25 \text{ m s}^{-1} \text{ east}$$

$$d \quad v_{\text{av}} = \frac{u+v}{2}$$

$$= \frac{5+25}{2}$$

$$= 15 \text{ m s}^{-1} \text{ east}$$

$$e \quad a = \frac{v-u}{t} = \frac{(25-5)}{10} = 2.0 \text{ m s}^{-2} \text{ east}$$

$$f \quad v_{\text{av}} = \frac{x}{t} = \frac{500}{50} = 10 \text{ m s}^{-1}$$

$$10 \text{ a } E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times (2000) \times (40.0)^2$$

$$= 1.6 \text{ MJ}$$

$$b \quad \Sigma F = ma$$

$$= m \frac{(v-u)}{t}$$

$$= 2000 \frac{(40.0-0)}{5.0}$$

$$= 2000 \times 8.0$$

$$= 1.6 \times 10^4 \text{ N}$$

$$c \quad \Sigma F = 16000 \text{ N} = F - 400 \text{ N}$$

$$\therefore F = 16.4 \text{ kN}$$

$$d \quad W = Fx = (1.64 \times 10^4) \times (100) = 1.64 \times 10^6 \text{ J} = 164 \text{ MJ}$$

$$e \quad P = \frac{W}{t} = \frac{1.64}{5.0} = 328 \text{ kW}$$

$$f \quad W = 400 \times 100 = 40 \text{ kJ}$$

$$g \quad \text{Efficiency} = \frac{1.6}{1.64} \times 100 = 97.6\%$$

$$11 \quad a \quad v^2 = u^2 + 2as$$

$$(8.0 \times 10^2)^2 = 0^2 + 2a \times 20$$

$$\therefore a = 1.6 \times 10^4 \text{ ms}^{-2}$$

$$b \quad F = ma = 550 \times 1.6 \times 10^4 = 8.8 \times 10^6 \text{ ms}^{-2}$$

$$c \quad p = mv = 550 \times 8.0 \times 10^2 = 4.4 \times 10^5 \text{ kg ms}^{-1}$$

d Momentum of shell = momentum of gun

$$(1.08 \times 10^5) \times v = 4.4 \times 10^5 \text{ kg ms}^{-1}$$

$$v = 4.1 \text{ ms}^{-1}$$

$$e \quad F = \frac{\Delta p}{\Delta t}$$

$$t = \frac{v - u}{a} = \frac{8.0 \times 10^2}{1.6 \times 10^4} = 0.05 \text{ s}$$

$$F = \frac{4.4 \times 10^5}{0.05} = 8.8 \times 10^6 \text{ N as before}$$

f Work = $F \times s$

$$= 8.8 \times 10^6 \times 20$$

$$= 1.8 \times 10^8 \text{ J}$$

$$g \quad E_k \text{ of shell} = \frac{1}{2}mv^2 = 1.8 \times 10^8 \text{ J}$$

This obviously represents an ideal situation; realistically there would be significant losses.

12 a



$$b \quad \lambda = \frac{2l}{5} = \frac{2 \times (1.50)}{5} = 0.600 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{(336)}{(0.600)} = 560 \text{ Hz}$$

$$c \quad \lambda = \frac{v}{f} = \frac{(336)}{(106)} = 3.17 \text{ m}$$

$$l = \frac{1\lambda}{2} = \frac{(3.17)}{2} = 1.59 \text{ m}$$

The second didgeridoo is longer.

13 a The average velocity over that distance.

b A radar gun only gives the instant velocity at a given point in time. Timing over 100 m gives an average velocity over that distance.

c The skier accelerates down the slope due to a component of gravity, $g \sin \theta$, acting down the plane. The frictional force of the skis against the snow acts against this and retards the acceleration.

d Air resistance also acts against the motion of the skier down the slope, resulting in them reaching a terminal velocity when this resistance is equal to $mg \sin \theta$.

e The design of the skis acts to reduce the force on any given part of the snow under the skis by spreading the weight of the skier and equipment over a larger surface area: pressure = $\frac{\text{force}}{\text{area}}$.

f

Design factor	How it reduces friction
shape of the skis	lower profile helps them to reduce wind resistance
helmet design	directs the wind from the top of their head down their back while in the tuck position
shape of the boots	low profile to lower wind resistance
skin-tight polyurethane suits	allows air to pass over the skier easily, reducing wind resistance

g Acceleration is the rate of change of velocity. The faster they accelerate, the quicker they will reach terminal speed.

$$\mathbf{h} \quad v = \frac{254.958}{3.6} = 70.8217 \text{ ms}^{-1}$$

$$v_{\text{av}} = \frac{s}{\Delta t} \text{ and } v_{\text{av}} = \frac{v+u}{2}$$

$$\frac{s}{\Delta t} = \frac{v+u}{2}$$

$$\begin{aligned} \Delta t &= \frac{2s}{v+u} \\ &= \frac{2 \times 340}{70.8217 + 0} \\ &= 9.60 \text{ s} \end{aligned}$$

$$\begin{aligned} \mathbf{i} \quad a &= \frac{v-u}{t} \\ &= \frac{70.82 - 0}{9.60} \\ &= 7.38 \text{ ms}^{-2} \end{aligned}$$

$$\begin{aligned} \mathbf{j} \quad v_{\text{av}} &= \frac{s}{\Delta t} \\ \Delta t &= \frac{s}{v_{\text{av}}} \\ &= \frac{100}{70.82} \\ &= 1.41 \text{ s} \end{aligned}$$

$$\begin{aligned} \mathbf{k} \quad E_{\text{k}} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times (70.0) \times (70.8217)^2 \\ &= 1.76 \times 10^5 \text{ J} \end{aligned}$$

$$\begin{aligned} \mathbf{l} \quad \mathbf{i} \quad E_{\text{k}} &= E_{\text{p}} \times \frac{93}{100} \\ E_{\text{p}} &= (1.76 \times 10^5) \times \frac{100}{93} \\ &= 1.89 \times 10^5 \text{ J} \end{aligned}$$

$$\begin{aligned} \mathbf{ii} \quad E_{\text{p}} &= mgh \\ h &= \frac{E_{\text{p}}}{mg} \\ &= \frac{(1.89 \times 10^5)}{(70.0) \times (9.80)} \\ &= 275 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Altitude} &= 2720 - 275 \\ &= 2445 \text{ m} \end{aligned}$$

$$\begin{aligned} \mathbf{m} \quad W_{\text{d}} &= \Delta E \\ F_s &= E_{\text{p}} - E_{\text{k}} \\ F &= \frac{E_{\text{p}} - E_{\text{k}}}{s} \\ &= \frac{(1.89 \times 10^5) - (1.76 \times 10^5)}{(340)} \\ &= 38.2 \text{ N} \end{aligned}$$